MAT 132

The average value of a function

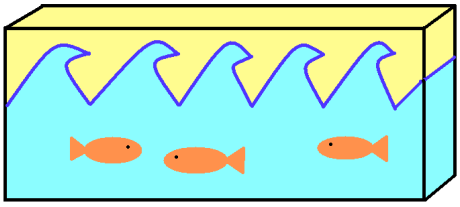
After explaining to a student through various lessons and examples that:

$$\lim_{x \to 8} \frac{1}{x-8} = \infty$$

I tried to check if she really understood that, so I gave her a different example. This was the result:

$$\lim_{x \to 5} \frac{1}{x-5} = \text{in}$$

Consider the following picture:



- How high would the water level be if the waves all settled?

The homework grades of a student are 6, 6, 7, 8, 10. Find the average homework score.

average = sum of grades/ number of hw

The temperature of a room is 70 degrees Fahrenheit at 10AM, 72 degrees Fahrenheit at 11:05AM and 74 at 11:30AM. Use these data to estimate the average temperature.

What if we want to make a more accurate estimation of the average temperature?

If the temperature is given by a function f, f(x)=temperature at time x, x in [a,b].
We want to estimate the average value of f.
Divide [a, b] into n equal intervals.

$$\Delta x = (b-a)/n$$

x_i is a number the i-th interval

We estimate for the average value:

$$f_{average} \approx \frac{f(x_1)+f(x_2)+\ldots+f(x_n)}{n}$$

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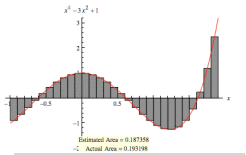
$$= \frac{\Delta x}{b-a}[f(x_1)+f(x_2)+\ldots f(x_n)]$$

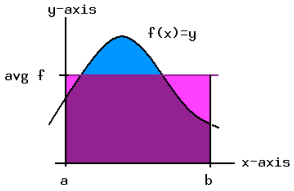
Since $\Delta x = (b-a)/n$

Taking limits $\frac{1}{b-a}\int_a^b f(x)dx$

Demo



Recall: In a Riemann sum, the area of each rectangle is (b-a)/n

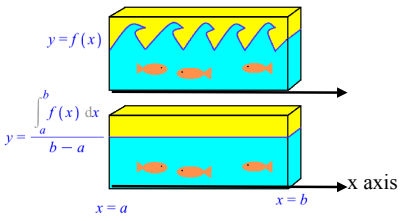


Example

- If f(x) = x^2, find the average value of f on the interval [1, 3] and interpret the result geometrically.

- http://www.calculusapplets.com/aveval.html
- Distance and Average Velocity for Piecewise Trajectory (Demo) http://demonstrations.wolfram.com/DistanceAndAverageVelocityForPiecewiseTrajectory/

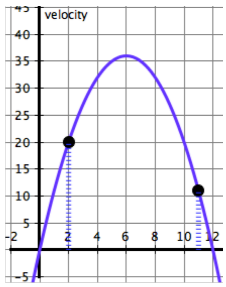
- How high would the water level be if the waves all settled?



- The temperature of a room is 70 degrees Fahrenheit at 10AM, 72 degrees Fahrenheit at 11:05AM and 74 at 11:30AM. Use these data to estimate the average temperature.
- The equation below gives the temperature T(t) of a room after t minutes. $T(t) = \frac{8}{14625}t^2 - \frac{14}{2925}t + 70$
- What is the average temperature during the first 90 minutes?
- What is the average temperature during the first 30 seconds?

The speed of an object is given by the equation $v(t) = 12t - t^2$ where v is in meters/sec and t is in seconds.

Determine the average speed of the object between t = 2 s and t = 11 s.

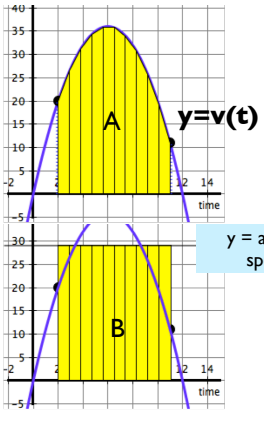


Area A= total distance

Two ways of visualizing the total distance (Area A and Area B)

Area B = average speed . time elapsed

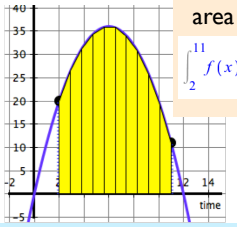
Average speed = total distance/ time elapsed



y = average speed

To determine the average value, we find a horizontal line such that the area under this horizontal line is equal to the area under the curve between two specified values of t.

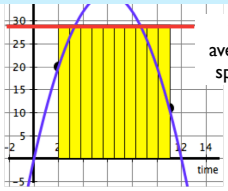
area = $\int_2^{11} f(x)\,dx$



Two ways of visualizing the total distance travelled.

area= average speed . time elapsed

Av speed = total distance/ time

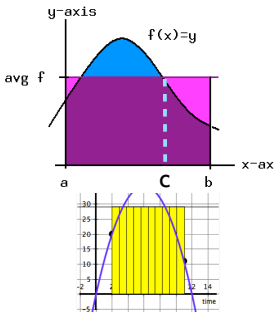


y = average speed

The mean value theorem for integrals

If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$



Find the average value of the function f(x)=sin(x) in the interval [0, π]. Also, find the smallest value of x at which the average occurs. Describe the geometric interpretation of the results.