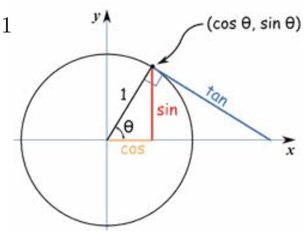
MAT 132

More Integration techniques:
Trigonometric integrals
Trigonometric substitution
Partial Fractions

Recall

$$\cos^2(x) + \sin^2(x) = 1$$



Example: Evaluate

$$\int \cos^3(x)dx$$

$$\int \sin^5(x)dx$$

These are examples of *trigonometric integrals*.
We are applying a special substitution.
sin(x)=u or cos(x)=u.

If possible, check your answer by differentiating the result

This method "works" when we have *odd powers of sin(x) or cos(x).*

Example: Evaluate

$$\int \cos^2(x)dx$$

$$\int \sin^4(x)dx$$

These are examples of *trigonometric integrals*.
We apply trigonometric identities:

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

This method "works" when we have *even powers of sin(x) or cos (x).*

Useful trigonometric formulae

$\cos^2 x + \sin^2 x = 1$ (and so, $1+\tan^2 x = \sec^2 x$)

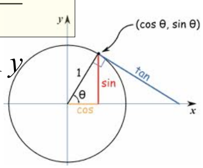
$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$$\text{then } \cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\text{and } \sin^2 x = \frac{1-\cos(2x)}{2}$$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$\tan x = \frac{\sin x}{\cos x}$$



Example: Evaluate

$$\int \sqrt{1-x^2} \qquad \int \sqrt{16-9x^2}dx$$

$$\int (1+u^2)^{-2}du \qquad \int (9+4u^2)^{-2}du$$

$$\int \sec(t)dt \qquad \int (z^2-1)^{-1/2}dz \qquad \int (z^2-25)^{-1/2}dz$$

Example: Evaluate

$$\int \sqrt{1-x^2} \qquad \int \sqrt{16-9x^2}dx$$

$$\frac{1}{2} x \sqrt{16-9x^2} + \frac{8}{3}\arcsin\left(\frac{3}{4}x\right)$$

$$\int (1+u^2)^{-2}du \qquad \int (9+4u^2)^{-2}du$$

$$\frac{1}{18}\frac{u}{9+4u^2} + \frac{1}{108}\arctan\left(\frac{2}{3}u\right)$$

$$\int \sec(t)dt \qquad \int (z^2-1)^{-1/2}dz \qquad \int (z^2-25)^{-1/2}dz$$

$$\frac{1}{2} z\sqrt{z^2-25} - \frac{25}{2}\ln\left(z+\sqrt{z^2-25}\right)$$

Example: Evaluate

$$\int \sqrt{1-x^2}dx \qquad \int \sqrt{9-t^2}dt$$

$$\int (9+4u^2)^{-2}du \qquad \int z^{-3}(z^2-9)^{-1/2}dz$$

$$\frac{1}{18}\frac{\sqrt{z^2-9}}{z^2} - \frac{1}{54}\arctan\left(\frac{3}{\sqrt{z^2-9}}\right)$$

$$\int \sqrt{1-x^2}dx \qquad \int \sqrt{a^2-b^2x^2}dx \qquad x = a\sin u/b$$

$$\int \sqrt{9+4x^2}dx \qquad \int \sqrt{a^2+b^2x^2}dx \qquad x = a\tan u/b$$

$$\int \sqrt{-25+16x^2}dx \qquad \int \sqrt{-a^2+b^2x^2}dx \qquad x = a\sec(u)/b$$

Evaluate, assuming that x>1

$$\int \frac{\sqrt{x^2-1}}{x^4}dx$$

$$\frac{1}{3}\frac{(x-1)(1+x)\sqrt{x^2-1}}{x^3}$$

Evaluate

$$\int \cos(x)\cos(2x)dx \qquad \int \frac{1}{x^2\sqrt{x^2+2}}$$

$$\frac{1}{2}\sin(x) + \frac{1}{6}\sin(3x) \qquad -\frac{\sqrt{1+x^2}}{x}$$

$$\int \sin(x)^3\cos(x)^3dz \qquad \int \frac{4-x^2}{x^2}dx$$

$$-\frac{1}{6}\sin(x)^2\cos(x)^4 - \frac{1}{12}\cos(x)^4$$

Find the area of the region bounded by the curve y=tan²(x), the x-axis and the line x=π/4

$\tan(x) - x$ $\qquad$ $1 - \frac{1}{4}\pi$