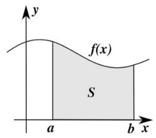
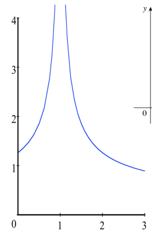
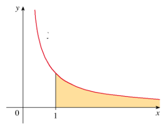
MAT 132

Section 5.10 Improper integrals

We have been work on definite integrals of <u>continuous</u> functions over <u>closed</u> intervals.







In certain cases, it is possible to find "areas under the curve" ("almost" definite integrals) of functions that have a point of discontinuity or integrals defined over infinite intervals

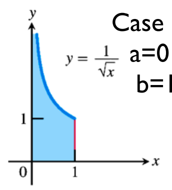
These are called <u>improper integrals</u>.

Infinite intervals Determine whether the integral is convergent or divergent. Evaluate if it is convergent.

$\int_1^\infty \frac{\ln(x)}{x^2}\,dx$	1	$-\frac{\ln(x)}{x} - \frac{1}{x}$
$\int_3^\infty \frac{1}{x}\,dx$		
$\int_{-\infty}^0 \frac{1}{x^2+4}\,dx$	$\frac{1}{4}\pi$	$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)$
$\int_{-\infty}^\infty \frac{1}{e^x+e^{-x}}\,dx$	$\frac{1}{2}\pi$	$\arctan(e^x)$

$$\int_a^b f(x)\,dx$$

1. when f is continuous in [a,b) and discontinuous at b.
2. when f is continuous in (a,b] and discontinuous at a.
3. when f is continuous in [a,c) and (c,b] for some c in [a,b]



Case 2.
a=0
b=1

Example: Evaluate the integral, if it is convergent.

$$\int_0^1 x\ln(x)\,dx$$

Use a comparison test to determine whether the integral is convergent or divergent.

$$\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx$$

A comparison test

If *f* and *g* are continuous functions with $0 \le f(x) \le g(x)$ For all $x \ge$ a. Then…..

$\int_a^\infty f(x)dx$ is convergent if $\int_a^\infty g(x)dx$ Converges

A function <u>converges</u> if its values are <u>smaller</u> than another function known to converge.

$\int_a^\infty g(x)dx$ is divergent if $\int_a^\infty f(x)dx$ Diverges

A function <u>diverges</u> if its values are <u>larger</u> than another function known to diverge.