

$$\int \arctan\left(\frac{1}{x}\right) dx$$

$$u = \arctan\left(\frac{1}{x}\right) \quad dv = dx$$

$$du = \frac{1}{1 + \left(\frac{1}{x}\right)^2} \cdot \left(-\frac{1}{x^2}\right) dx \quad v = x$$

$$= \frac{-1}{x^2 + x^2\left(\frac{1}{x^2}\right)} dx$$

$$= \frac{-1}{x^2 + 1} dx$$

$$= x \arctan\left(\frac{1}{x}\right) - \int \frac{-x}{x^2 + 1} dx$$

u-substitution

$$u = x^2 + 1$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$= x \arctan\left(\frac{1}{x}\right) + \int \frac{1}{u} \cdot \frac{1}{2} du$$

$$= x \arctan\left(\frac{1}{x}\right) + \frac{1}{2} \ln|u| + C$$

$$= x \arctan\left(\frac{1}{x}\right) + \frac{1}{2} \ln|x^2 + 1| + C$$