

$$\int \frac{x}{(2x+5)(x-2)} dx$$

$$\begin{aligned} \frac{x}{(2x+5)(x-2)} &= \frac{A}{2x+5} + \frac{B}{x-2} = \frac{A(x-2) + B(2x+5)}{(2x+5)(x-2)} \\ &= \frac{Ax + 2Bx - 2A + 5B}{(2x+5)(x-2)} = \frac{(A+2B)x + (-2A+5B)}{(2x+5)(x-2)} \end{aligned}$$

So $A+2B = 1 \Rightarrow A = 1-2B$ ↘ Plug into 2nd equation

$$-2A + 5B = 0$$

$$-2(1-2B) + 5B = 0$$

$$-2 + 4B + 5B = 0$$

$$9B = 2$$

$$B = \frac{2}{9}$$

$$A = 1 - 2B = 1 - \frac{4}{9} = \frac{5}{9}$$

So

$$\begin{aligned} \int \frac{x}{(2x+5)(x-2)} dx &= \int \frac{5/9}{2x+5} + \frac{2/9}{x-2} dx \\ &= \frac{5}{9} \int \frac{1}{2x+5} dx + \frac{2}{9} \int \frac{1}{x-2} dx = \frac{5}{18} \ln|2x+5| + \frac{2}{9} \ln|x-2| + C \end{aligned}$$