

$$\int_0^{\pi} e^x \sin(\pi-x) dx$$

$$u = \sin(\pi-x)$$

$$dv = e^x dx$$

$$du = -\cos(\pi-x) dx$$

$$v = e^x$$

$$= \underbrace{e^x \sin(\pi-x)} \Big|_0^{\pi} - \int_0^{\pi} -\cos(\pi-x) e^x dx$$

0 since  $\sin(\pi) = \sin(0) = 0$

$$= \int_0^{\pi} \cos(\pi-x) e^x dx$$

$$u = \cos(\pi-x)$$

$$dv = e^x dx$$

$$du = \sin(\pi-x) dx \quad v = e^x$$

$$= e^x \cos(\pi-x) \Big|_0^{\pi} - \underbrace{\int_0^{\pi} e^x \sin(\pi-x) dx}$$

$$\cos(0) = 1$$

$$\cos(\pi) = -1$$

This is what we started with...  
Call it A

$$\text{So } A = e^{\pi} \cos(0) - e^0 \cos(\pi) - A = e^{\pi} + 1 - A$$

$$\Rightarrow 2A = e^{\pi} + 1 \Rightarrow A = \frac{e^{\pi} + 1}{2}$$