

$$\int \cos^4(3x) dx$$

$$\text{use } \cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$$

$$\text{with } \theta = 3x$$

$$= \int [\cos^2(3x)]^2 dx = \int \left[ \frac{1}{2}(1 + \cos(6x)) \right]^2 dx$$

$$= \int \frac{1}{4} + \frac{1}{2} \cos(6x) + \frac{1}{4} \cos^2(6x) dx$$

use same formula

with  $\theta = 6x$

$$= \int \frac{1}{4} + \frac{1}{2} \cos(6x) + \frac{1}{4} \left[ \frac{1}{2}(1 + \cos(12x)) \right] dx$$

$$= \int \frac{1}{4} + \frac{1}{2} \cos(6x) + \frac{1}{8} + \frac{1}{8} \cos(12x) dx$$

$$= \frac{x}{4} + \frac{1}{12} \sin(6x) + \frac{x}{8} + \frac{1}{48} \sin(12x) + C$$

$$= \frac{3x}{8} + \frac{1}{12} \sin(6x) + \frac{1}{48} \sin(12x) + C$$