1. If p is a prime then $\sqrt{p}$ is irrational.

2. Let a and b be integers such that b is odd. Prove that if 6 divides $a.b$ then either 6 divides a or 3 divides b.

3. Find $\cap_{n\in\mathbb{N}}(-\frac{1}{n},\frac{1}{n})$ and $\cup_{n\in\mathbb{N}}(-\frac{1}{n},\frac{1}{n})$

4. Let Δ be a set and let $\{A_\alpha, \alpha \in \Delta\}$ be a family of sets. Determine which one of the following statements hold. If a statement is not true for every family, find a family for which is true. (You cannot use the empy set!) Assume that f is a function from A to A and each A_α is a subset of A.

   a $\cap_{\alpha\in\Delta}A_\alpha \subset \cup_{\alpha\in\Delta}A_\alpha$
   b $\cup_{\alpha\in\Delta}A_\alpha \subset \cap_{\alpha\in\Delta}A_\alpha$
   c $\cup_{\alpha\in\Delta}A_\alpha = \cap_{\alpha\in\Delta}A_\alpha$
   d $\cup_{\alpha\in\Delta}f(A_\alpha) = f(\cup_{\alpha\in\Delta}A_\alpha)$
   e $\cap_{\alpha\in\Delta}f^{-1}(A_\alpha) = f^{-1}(\cap_{\alpha\in\Delta}A_\alpha)$

5. Use mathematical induction to prove the following statment:. Suppose $a_1, a_2, ..., a_n$ are integers and p is a prime number. If $p|a_1a_2\cdots a_n$, then $p|a_i$ for some $i \in \{1, 2, ..., n\}$.

6. Prove that if a and b are are relatively prime, such that $a|m$ and $b|m$ then $a \cdot b|m$.

7. Prove that if x is a real number then $|x + 10| - |x - 1| \geq -11$.

8. Prove that $x^2 + y^2 \geq 2xy$ for all pairs of real numbers a and b.

9. Prove that if x is a positive real number then $x + 9/x > 6$.

10. Prove or disprove: $|n^2 - n + 17|$ is prime for all natural numbers n.

11. Prove by induction: $5n + 2 \leq n^2$ for all$n \geq 6$.

12. Prove by induction that for all natural numbers n, $n(n^2+5)$ is a multiple o 6.

13. Prove that for any real number x, $x + 20 > x$.

14. If $\mathcal{S}$ is a collection of sets and B is a set disjoint from every $A \in S$ then B is disjoint from $\cup_{A\in S}A$.

15. A collection of sets $\mathcal{S}$ is totally ordered if for every $A, B \in \mathcal{S}$ either $A \subset B$ or $B \subset A$. Let U be a set. Prove that a collection $\mathcal{S}$ of subsets of U is totally ordered if and only if the collection $S^c = \{U \setminus A : A \in \mathcal{S}\}$ is totally ordered.

16. Prove that the relation $\equiv$ on the set A is an equivalence relation if an only if it is reflexive and for every $a, b, c \in A$, if $a \equiv b$ and $a \equiv c$ then $b \equiv c$.

17. Prove that if f is a function from A to B and the family $\{B_i : i \in \Delta\}$ is a partition of B then $\{f^{-1}(B_i) : i \in \Delta\}$ is a partition of A.

18. Find an example of an injective function f from A to B and a surjective function g from B to C such that $f \circ g$ is not surjective.

19. Prove or disprove: If B is a finite set and $f : A \to B$ is surjective then A is finite.

20. Determine the cardinality of the following sets. (and prove your answer)

    a. $\{x \in \mathbb{R} : x > 0 \text{ and } x^2 \in \mathbb{Q}\}$
    b. The set of all multiples of 7.
    c. The set of irrational numbers.
    d. The set of all constant functions from $\mathbb{R}$ to $\mathbb{R}$
    e. The set of al finite subsets of $\mathbb{N}$ (this one is hard)

21. Prove that any two consecutive Fibonacci numbers are relatively prime.

22. Prove that if d is the greatest comon divisor of two numbers a and b then d^2 divides $a.b$.

23. Find all the elemens $[x]$ in $\mathbb{Z}_{12}$ such that $[x]^n = [0]$.

24. Find the sum of all elements of $\mathbb{Z}_n$ for all positive integers n.