MAT 141
Problem Set #3

due in recitation on September 23 or 24, 2004

1. Apostol, section I 3.12, # 1–4.
2. Prove the following theorem: If x is positive and $y > 1$, then $xy > x$.
3. Let $D = \{\frac{x+1}{x} \mid x > 1\}$. Show that D is both bounded above and bounded below. What are $\sup D$ and $\inf D$?
4. Let A be a set of real numbers which is bounded above and whose supremum is s. Define a new set $B = \{cx \mid x \in A\}$ where c is a positive real number. Prove that B is bounded above and that $\sup B = cs$.
5. Let A and B be non-empty sets of real numbers that are both bounded above, and define a new set $C = \{a - b \mid a \in A, b \in B\}$.
   (a) Show that the following statement is FALSE :
   $$\sup C = \sup A - \sup B$$
   by giving explicit examples of sets A and B for which the statement is not true.
   (b) If A and B are also bounded below, compute $\sup C$ in terms of $\sup A$, $\sup B$, $\inf A$, and $\inf B$. Prove your result.