HW9_ MAT322.

3-25; We integrate the characteristic function χ_{R_n}, where

$R_n = [a_1, b_1] \times \cdots [a_n, b_n] \subseteq \mathbb{R}^n$.

By Fubini's Thm;

$$\int_{\mathbb{R}^n} \chi_{R^n} = \int_{\mathbb{R}^{n-1}} \int_{\mathbb{R}} \chi_{R^n} = \underbrace{\int_{\mathbb{R}^{n-1}} \chi_{R^{n-1}}}_{\substack{\neq 0 \text{ by} \\ \text{induction} \\ \text{hypothesis}}} \underbrace{\int_{\mathbb{R}} \chi_{[a_n, b_n]}}_{\substack{\neq 0, \text{ as proved} \\ \text{before}}}$$

Note that we used the fact that $\chi_{R^n} = \chi_{R^{n-1}} \cdot \chi_{[a_n}$

But also note that we abused notation by calling both $\chi_{R^n} : \mathbb{R}^n \longrightarrow \{0, 1\}$ and $\chi_{R^{n-1}} : \mathbb{R}^{n-1} \longrightarrow \{0,$ by 'χ'.

3-28; $\cdot$ $D_{1,2}f$ and $D_{2,1}f$ being conti, so is $D_{1,2} - D_{2,1}f$.

Without loss of generality, assume $(D_{1,2} - D_{2,1})f(a) > 0$ (the case < 0 is of the same procedure). $\rightarrow \exists$ rectangle R around 'a'. s.t $(D_{1,2} - D_{2,1})f(x) > 0 \ \forall x \in R$. $R = [a, b] \times [c, d]$

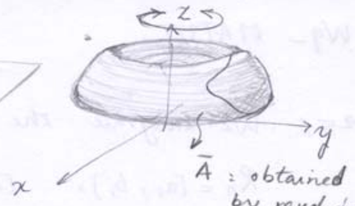
$$\int_R D_{1,2}f \overset{\text{Fubini's}}{=} \int_{x_2=c}^{d} \int_{x_1=a}^{b} D_{1,2}f = \int_{x_2=c}^{d} (D_2 f(b, x_2) - D_2 f(a, x_2)) dx_2$$

$$= f(b,d) - f(b,c) + f(a,c) - f(a,d).$$

Similarly: $\int_R D_{2,1}f = \int_{x_1=a}^{b} \int_{x_2=c}^{d} D_{2,1}f = \int_{x_1=a}^{b} [D_1 f(x_1, d) - D_1 f(x_1, c)]$

$= f(b,d) - f(a,d) - f(b,c) + f(a,c)$. This contradicts $\int_R (D_{2,1} - D_{1,2})f > 0$

3-28 (1) let $r=\sqrt{x^2+y^2}$.



$\bar{A}$: obtained by revolving A.

Then $\chi_{\bar{A}}(x,y,z) = \chi_A(\sqrt{x^2+y^2}, y)$.

$$\iiint \chi_{\bar{A}}(x,y,z)\,dx\,dy\,dz =$$

$$\iiint \chi_{\bar{A}} \cdot r\,dr\,d\theta\,dy = 2\pi \iint r\chi_A(r,y)\,dy\,dr = 2\pi\int r\left(\int \chi_A(r,y)\,dy\right)dr$$

3-31;

$$F(x_1,\dots,x_n) = \int_{[a^1,x^1]\times\cdots\times[a^n,x^n]} f \overset{\text{Fubini's Thm}}{=} \int_{[a^i,x^i]} \underbrace{\int_{[a^1,x^1]}\cdots\int_{[a^{i-1},x^{i-1}]}\int_{[a^{i+1},x^{i+1}]}\cdots\int_{[a^n,x^n]} f}_{g(x_1,\dots,x_{i-1},x_{i+1},\dots,x_n)}$$

$$\to D_i f \overset{\text{Fundamental Thm of calculus}}{=} \int_{[a^1,x^1]\times[a^2,x^2]}\cdots\int_{[a^{i-1},x^{i-1}]}\int_{[a^{i+1},x^{i+1}]}\cdots\int_{[a^n,x^n]} f$$

Fundamental Thm of calculus is applicable as $g(\cdot)$ is conti. (why?)