HW8 – MAT322.

3-14 – If f and g are discontinuous on Z_f & Z_g, $\Rightarrow$ $Z_{fg} \subseteq Z_f \cup Z_g$, where Z_{fg} is the set of discontinuities of 'fg':

As Z_f & Z_g are both of measure zero, so is $Z_f \cup Z_g$ and therefore Z_{fg}. $\Rightarrow fg$ is integrable.

3-18 – Let $E_n = \{x \mid f(x) > 1/n\}$.

By integrability, $\forall \varepsilon > 0$, $\exists P$: $\mathcal{U}(f,P) - \underset{= \int f}{0} < \varepsilon$:

Let $V_1^P, \ldots, V_N^P$ be the rectangles in P s.t $V_i^P \cap E_n \neq \emptyset$.

$\rightarrow$ $\varepsilon > \mathcal{U}(f,P) = \sum M_i v(V_i^P) \geq 1/n \sum v(V_i^P)$.

$\Rightarrow$ (*) $\sum v(V_i^P) < n\varepsilon$ and $E_n \subseteq \bigcup_i V_i^P$.

Therefore $\forall \varepsilon > 0$, $\exists$ a finite cover $\{V_i^P\}$ s.t (*) holds. $\Rightarrow$

E_n has content, and so measure, zero.

$E = \{x \mid f(x) \neq 0\} \subseteq \left(\bigcup_n E_n\right) \rightarrow$ of measure zero.

$\Rightarrow E$ is of measure zero. ▨

20 - $f : [a, b] \to \mathbb{R}$ increasing. $\to$ Number of points in

$E_n = \{x \mid \omega(x) > 1/n\}$ is less than $(f(a) - f(b))n$

That is E_n is finite $\to E = \bigcup E_n$ is countable $\to E$ is of measure zero.

22 - A is Jordan-measurable $\Rightarrow \overset{\text{boundary}}{\partial A}$ has measure zero.

$\to \forall \varepsilon > 0, \exists \overset{\text{rectangle}}{\{U_1, U_2, \dots\}}, \sum v(U_i) < \varepsilon : \partial A \subseteq \bigcup_i U_i$

Note that $\underset{\text{Boundary}}{\partial}(\overbrace{A \setminus \bigcup U_i}^{D}) \subseteq \partial \bigcup U_i \subseteq \bigcup \partial U_i \Rightarrow D$ is Jordan-measurable, for ∂U_i is of measure zero.

$\to C = \bar{D}$ is a compact set and $\int_{A \setminus C} 1 < \varepsilon$