W5 - MAT 322.

1- $x^2 - 3xy - y^2 - 2x + y - 5 = 0. \xrightarrow{\partial/\partial x} 2x - 3y - 3x\underset{\frac{dy}{dx}}{y'} - 2yy' - 2 + y' =$

$$\rightarrow y' = \frac{2 - 2x + 3y}{1 - 2y - 3x}.$$

2- $\cos x + Tg(xy) + 5 = 0.$

$\frac{\partial}{\partial x}: \quad -\sin x + (1 + Tg^2(xy))(xy' + y) = 0$

$$\Rightarrow y' = \frac{\sin x - y(1 + Tg^2(xy))}{x}.$$

3- $\ln\sqrt{x^2+y^2} + xy = 4$

$\frac{\partial}{\partial x}: \quad \frac{1}{2}(2x + 2yy')\frac{1}{x^2+y^2} + y + xy' = 0$

$$\Rightarrow y' = \frac{-y(x^2+y^2) - x}{x(x^2+y^2)+y}$$

4- $x - y^2(x^2+y^2) = x - x^2y^2 - y^4 = 6x^2 + 6y^2$

$\rightarrow -x + 6x^2 + x^2y^2 + 6y^2 + y^4 = 0$

$\partial/\partial x \rightarrow -1 - 12x + 2xy^2 + 2yy'x^2 + 12yy' + 4y'y^3 = 0$

$$y' = \frac{12x + 1 - 2xy^2}{2yx^2 + 12y + 4y^3}$$

$-, x^2+y^2+z^2=25.$

Note that by implicit function theorem 'z' can be con
as a function of x & y ; ie, $z=z(x,y)$.

$$\to \frac{\partial}{\partial x} : \frac{\partial}{\partial x}(x^2) + \underbrace{\frac{\partial}{\partial x}(y^2)}_{0} + \frac{\partial}{\partial x} z^2 = 0$$

x, y are independent coordinates.

$$2x + 0 + 2\frac{\partial}{\partial x} z \cdot z = 0$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{x}{z}$$

similarly

$$\frac{\partial z}{\partial y} = -\frac{y}{z}.$$

$; - Tg(x+y) + Tg(y+z) = 1.$

$$\frac{\partial}{\partial x} : (1+0)\left(1+Tg^2(x+y)\right) + \left(0+\frac{\partial z}{\partial x}\right)\left(1+Tg^2(x+z)\right) = 0$$

$$\frac{\partial z}{\partial x} = -\frac{1+Tg^2(x+y)}{1+Tg^2(x+z)}$$

similarly; $\frac{\partial}{\partial y} : (0+1)(1+Tg^2(x+y)) + \left(1+\frac{\partial z}{\partial y}\right)\left(1+Tg^2(y+z)\right) = 0$

$$\frac{\partial z}{\partial y} = -\frac{2+Tg^2(x+y)+Tg^2(y+z)}{1+Tg^2(y+z)}$$

$- e^{xz} + xy = 0$

$$\left(\frac{\partial z}{\partial x} \cdot x + z\right)e^{xz} + y = 0.$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{-y - ze^{xz}}{xe^{xz}}$$

$$\left(\frac{\partial z}{\partial y} x + 0\right)e^{xz} + x = 0.$$

$$\frac{\partial z}{\partial y} = -\frac{x}{xe^{xz}}$$

$1-\quad x\ln y + y^2 z + z^2 = 8$

$\frac{\partial}{\partial x}:\quad \ln y + y^2 \frac{\partial z}{\partial x} + 2z\frac{\partial z}{\partial x} = 0$

$$\rightarrow \frac{\partial z}{\partial x} = -\frac{\ln y}{y^2+z}$$

$\frac{\partial}{\partial y}:\quad \frac{x}{y} + 2yz + y^2\frac{\partial z}{\partial y} + 2z\frac{\partial z}{\partial y} = 0$

$$\frac{\partial z}{\partial y} = -\frac{x+2y^2z}{y^3+2yz}$$

$1-\quad xyz + xzw - yzw + w^2 = 5$

$w = w(x,y,z)$

$-yz + zw - 0 + 2w\frac{\partial w}{\partial x} = 0 \Rightarrow \frac{\partial w}{\partial x} = -\frac{yz+zw}{2w}$

$-xz + 0 - zw + 2w\frac{\partial w}{\partial y} = 0$

$$\Rightarrow \frac{\partial w}{\partial y} = \frac{zw - xz}{2w}$$