MAT322;
HW1.

1; $|x|^2 = \sum^n x_i^2 \le \sum_{i,j=1}^n |x_i||x_j| = \left(\sum_{i=1}^n |x_i|\right)^2$

$\rightarrow |x| \le \sum_{i=1}^n |x_i|$ ■

-2; 1-7-2 gives the necessary condition of linear depen

If $x \neq 0 \Rightarrow y = \lambda x$ for $\lambda \in \mathbb{R}$.

$\rightarrow \underbrace{\langle x, y \rangle}_{\text{inner product}} = \sum x_i y_i = \lambda \sum x_i^2$

$|x||y| = |\lambda||x|^2$

$\left.\right\} \rightarrow \lambda = |\lambda| \Rightarrow \lambda \ge 0$ ■

1-3; We know that $|x + (-y)| = |x-y| \le |x| + |y| = |x| + |-y|$
And the equality holds when $x = \lambda(-y) = \tilde{\lambda} y$, $\tilde{\lambda} \le 0$.

1-4; $|x| = |x-y+y| \le |x-y| + |y| \rightarrow |x| - |y| \le |x-y|$.
the argument by exchanging x & y $\rightarrow |y| - |x| \le |x-y|$ $\Big\} \Rightarrow$

$\big||x| - |y|\big| \le |x-y|$ ■

1-5; $|z-x| = |z-y+y-x| \le |z-y| + |y-x|$.

Distance between two points is the length of the line segm
connecting them. $\Rightarrow$ The length of a side of a trian
is less than summation of the lengths of other two sides

1-6; a) $(f-\lambda g)^2 \ge 0 \rightarrow \int_a^b (f-\lambda g)^2 dx \ge 0 \rightarrow \int_a^b f^2 - 2\lambda \int_a^b f \cdot g \, dx + \int_a^b g^2 \ge 0$

If both $\int f^2 = \int g^2 = 0 \rightarrow \forall \lambda \in \mathbb{R}\ \lambda \int f \cdot g \ge 0 \rightarrow \int f \cdot g = 0$: noth

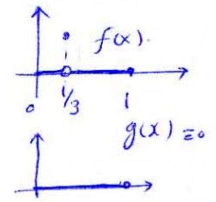
Without loss of generality, let $\int g^2 \neq 0$

Let $\lambda = \int fg \Big/ \int g^2 \Rightarrow \int f^2 - \dfrac{(\int fg)^2}{\int g^2} \ge 0 \Rightarrow \int f^2 \int g^2 \ge (\int fg)$

b) Not necessarily; unless $f-\lambda g$, for $\lambda = \int fg \big/ \int g^2$, is continuous (In special case, when both f & g are conti.)

Remark; If $\psi : [a,b] \to \mathbb{R}^{\geq 0}$ is conti $\to$

$$\int_a^b \psi \, dx = 0 \longleftrightarrow \psi \equiv 0$$



$\int (f-\lambda g)^2 = 0$, but $\underline{\underline{\text{no}}}$ λ : $f = \lambda g$.

c) Define : $f : [0, n] \to \mathbb{R}$, $f(t) = x_i \quad i-1 \leq t \leq i$

$g : \quad " \to " \quad , g(t) = y_i \quad i-1 \leq t \leq i$

$$\left.\begin{array}{l} \to \int fg = \langle x, y \rangle \\ \int f^2 = |x|^2 \quad , \int g^2 = |y|^2 \end{array}\right\} \to \langle x, y \rangle \leq |x||y|. \quad \blacksquare$$