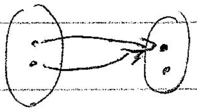
Sol'ns

a .

b

P	$\sim P$	$P \to \sim P$
T	F	F
F	T	T

neither taut, nor contrad.

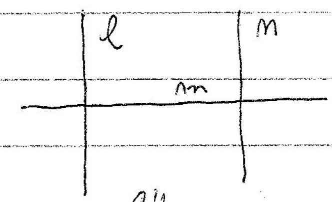
c Not for every possible example, hence not true:
$U \neq \emptyset \quad A = \emptyset \quad B = U$: $A \cap B' = \emptyset \cap \emptyset = \emptyset$; $A' \cup B = U \cup U = U$; $U \neq \emptyset$.

d If $A \neq \emptyset$, then $A \times \emptyset = \emptyset \neq A$. If $A = \emptyset$: $A \times \emptyset = \emptyset = A$.
Answer: iff $A = \emptyset$.

e No: equivalent to $\exists x\, P(x) \wedge \sim \forall x\, P(x)$, $\exists x\, P(x) \wedge \exists x \sim P(x)$, and if $P(x)$, for example, is a contradiction, then $\forall x \sim P(x)$, so $\exists x \sim P(x)$ but $\exists x\, P(x)$ false.

f $\exists x \cancel{\sim\sim} \exists y \sim \forall z\, (P \wedge \sim Q)$, $\exists x,y,z \sim (P \wedge \sim Q)$,
$\exists x,y,z\, (P \to Q)$

g No $A-B-C$ means $\forall f$: $f(A) < f(B) < f(C)$ or
$f(C) < f(B) < f(A)$.

h No  , false in $\mathbb{R}^2$ plane.

$\ell \parallel m$

$\boxed{1}$ By axioms: $x=0$ no inverse: else $x \cdot y = 1$, but $0 \cdot y = 0 \ \forall y$;
$x \neq 0$ has an inverse.

So x has a inverse IFF $x \neq 0$.

Let x have inverses say y and y': $x \cdot y = 1$ $x \cdot y' = 1$.

Then $x \cdot y - x \cdot y' = 0$; $x(y-y') = 0$; ~~$x \cdot y$~~ $y \cdot (x(y-y')) = 0$;

$(y \cdot x)(y-y') = 0$; $y - y' = 0$; $y = y'$: inverse is unique.

$\boxed{2}$ First show $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ by induction. (See book; I will not repeat it here).

Then $\sum_{i=4}^{n} = \sum_{i=1}^{n} - 1 - 2 - 3 = \frac{n(n+1)}{2} - 6 = \frac{n^2+n}{2} - 6 = \ldots$

$\boxed{3}$ n is a multiple of k; $\exists \tilde{m} \ (n = \tilde{m}k)$ (Premise)

$n^2 = n \cdot n = n \cdot (\tilde{m}k) = (n\tilde{m})k$.

Hence $\exists n\tilde{m} \ ((n\tilde{m})k = n^2)$ that is n^2 is a multiple of k.

$\boxed{4}$ $\sim R$ and $\sim R \to \sim Q$ give $\sim Q$.

$(P \to Q) \wedge \sim Q$ give $\sim P$ as follows: $P \to Q \wedge (\sim Q) \wedge (P \vee \sim P)$;

By cases: $P \wedge (P \to Q) \wedge (\sim Q)$ gives $Q \wedge \sim Q$ contradiction.

by indirect proof, P leads to contradiction, so $\sim P$.

$\boxed{5}$ Assume ~~the conditional~~ f is 1-1.

$f \circ g = f \circ h$ means $\forall x \ f(g(x)) = f(h(x))$.

Since f is 1-1 $\forall x$ $g(x) = h(x)$ so $g = h$.

Assume the conditional. Indirect proof: assume f not 1-1.

So $\exists x_1, x_2$ with $f(x_1) = f(x_2)$, but $x_1 \neq x_2$.

Let $C = \{1\}$ $g: 1 \to x_1$ $h: 1 \to x_2$. Then $g \neq h$, but

$f(g(1)) = f(x_1) = f(x_2) = f(h(1))$. Contradiction.

$\boxed{6}$ $\quad T \quad S \quad R$

$\quad A\times B \quad B\times C \quad C\times D$

$R\circ S = \{(b,d) \mid \exists c\,(bSc \wedge cRd)\}$

$(R\circ S)\circ T = \{(a,d) \mid \exists b\,(aTb \wedge b\,(R\circ S)\,d\}$

$= \{(a,d) \mid \exists b\,(aTb \wedge \exists c\,(bSc \wedge c.Rd)\}$

$= \{(a,d) \mid \exists b,c\,(aTb \wedge bSc \wedge cRd)\}$

$\cancel{=\{(a,d) \mid \exists b\,(aTb \wedge bSc) \wedge}$

$R\circ(S\circ T) = \{(a,d) \mid \exists c\,(a\ S\circ T c \wedge cRd)\}$

$= \{a,d \mid \exists c\,(\exists b\ \underline{aTb} \wedge \underline{bSc} \wedge \underline{cRd})\}$

$\boxed{7}$ By incidence axiom: if at least 2 pts the 2 transverse lines would be 2 <u>distinct</u> lines thru 2 distinct points, violating the axiom.

$\boxed{8}$ $A-B-C$ IFF $\exists f: l\to\mathbb{R}$ $\ f(A)<f(B)<f(C)$. (1)

$B-A-C$ IFF $\exists g: l\to\mathbb{R}$ $\ g(B)<g(A)<g(C)$ (2)

$g = \begin{cases} f+c & \Rightarrow (1) \Rightarrow g(A)<g(B)<g(C) \ \to\!\leftarrow (2) \\ -f+d & \Rightarrow (1) \quad g(C)<g(B)<g(A) \ \to\!\leftarrow (2). \end{cases}$

$\boxed{9}$ Assume D in interior. Then $D\notin\overleftrightarrow{AB}$, $D\notin\overleftrightarrow{AC}$.

Also D & B same side of $\overleftrightarrow{AC}$ so $\overline{DB}\cap\overleftrightarrow{AC}=\emptyset$

Similarly $\overline{DC}\cap\overleftrightarrow{AB}=\emptyset$.

Conversely: $D\notin\overleftrightarrow{AB}$ & $D\in\overleftrightarrow{AC}$ so D not on the lines. $\overline{DB}\cap\ldots$ etc precisely says that D interior.