SAMPLE FINAL

Questions a, ..., h ; 8x10 = 80 pts. Give brief justification.
Problems 1, ..., 4 ; 4x30 = 120 pts. Give proofs.
Problems 5, ..., 9 ; 5x30 = 150 pts. Give proofs.
= 350 pts

~ o ~

a Give an example of a function which is neither 1-1, nor onto.

b Is $P \to \sim P$ a contradiction?

c Given sets $A \subseteq U$, $B \subseteq U$, define $A' = U - A$, $B' = U - B$.
Is $A \cap B' = A' \cup B$?

d When is $A = A \times \phi$?

e Is $\sim(\exists x P(x) \to \forall x P(x))$ a law of logic?

f Simplify: $\sim \forall x \sim \exists y \sim \forall z (P \wedge \sim Q)$.

g Let A, B, C be on a line l and be such that A–B–C.
Is there a coordinate system f on l such that: $f(B) = 1$, $f(A) = 2$, $f(C) = 3$?

h If l and m are transverse lines, and m and n are transverse lines, then l and n are transverse.
Is the above always true?

$\boxed{1}$ Use axioms for $\mathbb{R}$ to show that if a multiplicative inverse to $x \in \mathbb{R}$ exists, then it is unique.

$\boxed{2}$ Show that $\sum_{i=4}^{n} i = \frac{n^2}{2} - 6 + \frac{n}{2}$.

$\boxed{3}$ Use quantifiers rigorously to show that if n is a multiple of k, then so is n^2.

$\boxed{4}$ Is the following argument valid:

$$[(P \to Q) \wedge (\sim R \to \sim Q) \wedge \sim R] \Rightarrow \sim P \quad ?$$

$\boxed{5}$ Prove that a function $f : A \to B$ is 1-1 if and only if $\forall g, h : C \to A \; [(f \circ g = f \circ h) \Rightarrow g = h]$.

$\boxed{6}$ Prove that, when composable: $(R \circ S) \circ T = R \circ (S \circ T)$ (associativity of relations)

$\boxed{7}$ Prove that the intersection of 2 transverse lines consists of exactly one point.

$\boxed{8}$ Prove that for 3 distinct points on a line, one cannot have $(A-B-C) \wedge (B-A-C)$ (May use any theorem in notes).

$\boxed{9}$ If $\angle BAC$ is not straight, prove that D is in the interior if and only if

$$D \notin \overleftrightarrow{AB} \wedge D \notin \overleftrightarrow{AC} \wedge \overline{DB} \cap \overleftrightarrow{AC} = \emptyset$$
$$\wedge \overline{DC} \cap \overleftrightarrow{AB} = \emptyset$$