EXAMPLES OF PROOFS

THM A-2 p 379 : There is a unique multiplicative identity

Proof (We use axioms V p. 374-5)

EXISTENCE Axiom V.9 states $\exists\, 1$.

UNIQUENESS Say 1 & y are 2 multiplicative identities; we must show $1 = y$.

We have $1 \cdot y = y$ (1 is identity).

We also have $1 \cdot y = 1$ (y is identity).

We get $y = 1$ $\boxed{}$

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THM A-14 p 384 There is no largest #.

[May assume $1 > 0$]

Proof. We must prove $\sim \exists x\, \forall y\, (x \geq y)$.

By De Morgan, same as $\forall x\, \exists y\, (x < y)$.

By V.16 $x + 1 > x + 0 = x$ so $x + 1 > x$.

Set $y := x + 1$.

Then $x < y$.

$\boxed{}$

TUM A-15.a p 384 $|xy| = |x||y|$.

Proof. By cases: 1) either x or y is zero;
2) neither is zero.

CASE 1 If $x=0$, then $xy=0$, so $|xy|=0$; also $|x|=0$, so $|x||y|=0$.
Hence: if $x=0$, then $|xy|=|x||y|$.
Same if $y=0$.

CASE 2 There are 4 subcases:
i) $x>0, y>0$; ii) $x>0, y<0$; iii) $x<0, y>0$; iv) $x<0, y<0$.

i) $x>0, y>0$. $x \cdot y > 0 \cdot y = 0$ (K17). So $|xy| = \cancel{|x||y|}\ xy$.
$|x| \cdot |y| = x \cdot y$.
Hence $|xy| = |x||y|$.

ii) $x>0, y<0$. $y \cdot x < 0 \cdot x = 0$ (K17). So $|y \cdot x| \overset{K.5}{=} |xy| = -xy$.
$|x| = x$
$|y| = -y$ done in class
$|x| \cdot |y| = x \cdot (-y) \overset{*}{=} -xy$
Hence $|xy| = |x||y|$.

iii & iv similar. $\boxtimes$