MIDTERM 2

1 Use induction to prove that: $\forall n \geq 2 \; (n+1 < 2^n)$.

2 Prove, using axioms V, that $(-1)^2 = 1$.
(you may use, without proof, that additive inverses $\exists!$)

3 Re-write the set $\{x \in \mathbb{R} \mid x^4 - 1 = 0 \text{ and } x \geq 1/2\}$ in interval notation.

4 For sets $A \subseteq B \subseteq C$, prove $C - A = (C-B) \cup (B-A)$.

5 Let $S \subseteq \mathbb{R} \times \mathbb{R}$ be the relation defined by $x S y$ IFF $x = \sin y$.

(5.i) Is S a function? Why?

(5.ii) Is S^{-1} a function? Why?

6 Let $T = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid 4x^2 - y^2 = 0\}$.

(6.i) Graph T.

(6.ii) Graph T^{-1}.

$\underline{1}$ $P(2)$: $2+1 < 2^2$ or $3<4$ ok.

Assume $P(n)$: $n+1 < 2^n \Rightarrow n+1+1 < 2^n+1 \Rightarrow (n+1)+1 < 2^n+2 = 2^{n+1}$. ∎

$\underline{2}$ $-1\cdot 0 = 0$; $-1(1+(-1))=0$; $-1\cdot 1+(-1)(-1)=0$; $-1+(-1)^2=0$.

$1+(-1)=0$ says 1 is the unique additive inverse to -1 so $(-1)^2=1$

$\underline{3}$ $x^4-1=(x^2-1)(x^2+1)\le 0$ IFF $x^2-1\le 0$ (since $x^2+1\ge 0$ always).

IFF $x\in[-1,1]$

$[-1,1]\cap[\frac{1}{2},\infty) = \cancel{[\frac{1}{2},1]}$ $[\frac{1}{2},1]$.

$\underline{4}$ i) $C-A \subseteq C-B \cup B-A$: $x\in C-A$ IFF $x\in C$ & $x\notin A$

By cases: $x\notin B$, $x\in B$. If $x\notin B$, then $x\in C$ & $x\notin B \Rightarrow x\in C-B$.

If $x\in B$, then $x\in B$ & $x\notin A \Rightarrow x\in B-A$.

ii) $C-A \supseteq C-B\cup B-A$.

By cases: $x\in C-B$ & $x\in B-A$.

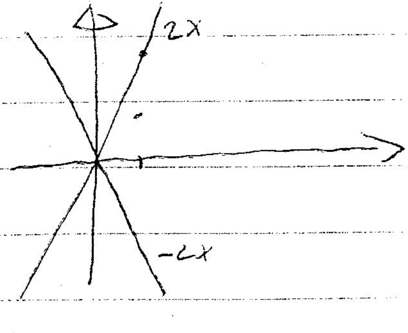
If $x\in C-B$, then $x\in C$ & $x\notin A$ so $x\in C-A$.

If $x\in B-A$, then $x\in C$ & $x\notin A$ so $x\in C-A$.

5) (5.i) Not a function: $(0,0),(0,2\pi)\in S$, so 0 in the first place corresponds to 0 & 2π in the second place.

(5.i) S^{-1} is $y=\sin x$, a function.

6) $4x^2-y^2=0$ $(2x-y)(2x+y)$: T:



$T^{-1}=\{(x,y) \mid 4y^2-x^2=0\}$ $(2y-x)(2y+x)=0$

T^{-1}: