MIDTERM I

1) The domain for x, y, z is $\mathbb{R}$. Determine if true or false.

1.a) $\forall x \forall y \exists z \, (x^2+y^2=z^2)$

1.b) $\forall x \forall y \exists! z \, (x^2+y^2=z^2)$

1.c) $\forall x \exists y \forall z \, (-x^2+y^2=z^2)$

1.d) $\exists x \exists y \forall z \, (x^2+y^2 \neq z^2)$

2) Use a truth table to determine whether $P \rightarrow (Q \rightarrow \sim P)$ is a tautology, a contradiction, or neither.

3) Show, without truth tables, the validity of:
$(Q \wedge \sim P) \rightarrow (Q \vee P) \wedge (P \rightarrow Q)$

4) Translate, using propositional variables, connectives, quantifiers, etc.:
"THERE ARE PEOPLE WHO ALWAYS BREATHE, BUT SOMETIMES DO NOT SMOKE."
(Atomic statements only!)

5) Is the following correct?

5.a)

Premise: $P \rightarrow Q$ false.
Hence, $\sim P \rightarrow \sim Q$.
Taking contrapositives $Q \rightarrow P$.
Conclusion: $Q \rightarrow P$ true.

5.b) Give an example of $\sim R \rightarrow \sim T$ false, but $R \rightarrow T$ true.

6) Find a domain in $\mathbb{R}$ for the free variable such that:
$\forall x \, (x > y \rightarrow x^2 > y^2)$ is false

1a TRUE : $z = \pm\sqrt{x^2+y^2}$

1b FALSE : z is not unique, for example $z = \pm\sqrt{1^2+0^2} = \pm 1$

1.c. FALSE : $y = \pm\sqrt{x^2+z^2}$ $z=0$ $y = \pm|x|$, $z=1$ $y = \pm\sqrt{x^2+1}$ not same y for all z.

1.d FALSE : $\sim$(1.a) & 1.a is TRUE.

2.

P	Q	Q→∼P	P→(Q→∼P)
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Neither

3. $Q \wedge \sim P \to Q$; $Q \to Q \vee P$. Hence $Q \wedge \sim P \to (Q \vee P)$.

$Q \wedge \sim P \to \sim P$; $\sim P \to \sim P \vee Q$ but $(\sim P \vee Q) \Leftrightarrow \sim\sim(\sim P \vee Q) \Leftrightarrow \sim(P \wedge \sim Q)$

$\Leftrightarrow P \to Q$.

So $(Q \wedge \sim P) \to [(Q \vee P) \wedge (P \to Q)]$

4. p variable, domain = people on earth. B(p,t) is p breathes at time t

S(p,t) is p smokes at time t. $\exists p [(\forall t\, B(p,t)) \wedge \exists t \sim S(p,t))]$

5.a No : $\sim(P \to Q)$ is not $\sim P \to \sim Q$.

5.b R = it rains T = the streets are wet

6 y in $(-\infty, 0)$ Then. for example if $x > y$ then $-1 > -2$; not true that $1 > 4$.