If you use the max-min inequality to find upper and lower bounds for integrals you find

$$0.5 \leq \int_0^1 1/(1+x^2) \leq 1.$$

Use the fact that you can break the interval $[0,1]$ into the two parts $[0,0.5]$ and $[0.5,1]$ to find an improved estimate

$$???? \leq \int_0^1 1/(1+x^2) \leq ???.$$

Solution. The function $f = 1/(1+x^2)$ is monotonic decreasing on $[0,1]$.
$max\, f = 1$ on the interval $[0,0.5]$
$min\, f = 4/5$ on the interval $[0,0.5]$.
The min-max inequality on $[0,0.5]$ gives

$$2/5 \leq \int_0^{0.5} 1/(1+x^2) \leq 1/2.$$

$max\, f = 4/5$ on the interval $[0.5,1]$
$min\, f = 1/2$ on the interval $[0.5,1]$.
The min-max inequality on $[0.5,1]$ gives

$$1/4 \leq \int_{0.5}^1 1/(1+x^2) \leq 2/5.$$

Putting together

$$13/20 = 2/5+1/4 \leq \int_0^{0.5} 1/(1+x^2)\,dx + \int_{0.5}^1 1/(1+x^2)\,dx = \int_0^1 1/(1+x^2)\,dx \leq 1/2+2/5 = 9/10.$$