8.1 4b (b). From (a). $\lim_{n\to\infty} a_n = L$. I Oct. 1/07

$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{2+a_n}$, by limit law we have

$L = \sqrt{2+L}$, i.e. $L^2 - L - 2 = 0$ and $L \geq 0$.

Hence $L = 2$. (The other root $L = -1$ is ruled out by $L \geq 0$).

8.2. 12 $\sum_{n=1}^{\infty} (0.4)^n = \frac{1}{1-0.4} = \frac{5}{3}$

28 $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3} \right)$

$$= \lim_{k\to\infty} \Big(\frac{1}{2} + 0 - \cancel{\frac{1}{4}} + \frac{1}{3} + 0 - \cancel{\frac{1}{5}} + \cancel{\frac{1}{4}} + 0 - \cancel{\frac{1}{6}} + \cancel{\frac{1}{5}} + 0 - \cancel{\frac{1}{7}} + \cdots + \cancel{\frac{1}{k-1}} + 0 - \cancel{\frac{1}{k+1}} + \cancel{\frac{1}{k}} + 0 - \frac{1}{k+2} + \cancel{\frac{1}{k+1}} + 0 - \frac{1}{k+3} \Big)$$

$$= \lim_{k\to\infty} \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{k+2} - \frac{1}{k+3} \right) = \frac{5}{6}$$

II.

41. $S_n = \frac{n-1}{n+1}$

$a_1 = S_1 = 0$

$a_n = S_n - S_{n-1} = \frac{n-1}{n+1} - \frac{n-2}{n} = \frac{2}{n(n+1)} \quad n \geq 2$

$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n-1}{n+1} = 1$

52. $\sum a_n$ $\sum b_n$ are both divergent.

$\sum (a_n + b_n)$ could be convergent.

ex. $\sum a_n = \sum_{n=1}^{\infty} \frac{1}{n}$ $\quad \sum b_n = \sum_{n=1}^{\infty} \frac{-1}{n+1}$

$\sum (a_n + b_n) = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right) = 1$

8.3 28c. It is easy to check $\frac{1}{x^4}$ is positive, decreasing for $x>0$.

So we can use the following formula:

$$\underline{R_n \leq \int_n^{\infty} \frac{1}{x^4} dx} = -\frac{x^{-3}}{3}\Big|_n^{\infty} = \frac{n^{-3}}{3} \leq 10^{-5},$$

Then solve $n^3 \geq \frac{10^5}{3}$, we get $n \geq 10 \sqrt[3]{\frac{100}{3}}$

84 33 (a) (d) [I omit the details, please check that for these 2 (only), $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$]

13 $|R_n| \leq b_{n+1} = \frac{1}{(n+1)^6} < 0.00005$

then $n > (2\times 10^4)^{\frac{1}{6}} - 1$

Please use your calculator to get the final results of 28c and 13.

Which among 26 28 29 23 is absolutely Convergent? III.

26. $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$ is absolutely convergent.

Since $|a_n| \le \frac{1}{4^n}$ and $\sum_{n=1}^{\infty} \frac{1}{4^n}$ is convergent,

28 By ratio test, $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{5^n}{(n+2)^2 4^{n+3}} \cdot \frac{4^{n+2}(n+1)^2}{5^{n-1}} = \frac{5}{4} > 1$

it diverges.

29. By ratio test $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{1\cdot3\cdot5\cdots(2n-1)(2n+1)}{(2n+1)!} \cdot \frac{(2n-1)!}{1\cdot3\cdot5\cdots(2n-1)}$

$$= \lim_{n\to\infty} \frac{1}{2n} = 0 < 1$$

so it is absolutely convergent.

23. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} > \sum_{n=1}^{\infty} \frac{1}{n}$, this tells us it is not absolutely convergent.

$\frac{1}{\sqrt{n}}$ $(n \geq 1)$ is decreasing and converge to 0 as $n \to \infty$,

this tells us $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ converges.

31 $\frac{a_{n+1}}{a_n} = \frac{5n+1}{4n+3}$ $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{5}{4} > 1$ $\therefore \sum a_n$ diverges by ratio test.