MAT 141 - Fall 2004

Final Exam

December 17, 2004

Name: ______________________________

ID#: ______________________________

Instructions: The exam consists of 10 questions. You have 150 minutes to answer all the questions. Show all your work. You are not allowed to use any books, notes, or calculators. Good luck.

Warning: Do not discuss the contents of this exam with anyone until all students have taken it. Providing another student with advance knowledge of the exam is a violation of the university's academic honesty policy and will be reported to the Office of Academic Judiciary.

Problem	Max	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 points) Compute the derivative of each of the following functions.

a. $f(x) = x^3 + 7x - 4$

b. $g(x) = \sin(\cos(\sin x))$

c. $A(x) = \sin x \cos x \sin x$

d. $w(x) = [x]$

e. $r(t) = \dfrac{\sqrt{t} + 1}{\tan t}$

f. $I(x) = \displaystyle\int_0^{x^2} (t^2 + 1)\, dt.$

2. (10 points) Compute the area between the graphs of $y = \sin^2 x$ and $y = \cos^2 x$ over the interval $[0, \pi/2]$.

3. (10 points) In order to get a job at the nuclear power plant, Homer Simpson had to learn calculus. Unfortunately, he never understood it. All he remembers is that the "d'oh–rivitive" of a function, $f(x)$, written $f^*(x)$, satisfies the following rules:

- If $f(x) = c$, then $f^*(x) = 0$.
- If $f(x) = x$, then $f^*(x) = 1$.
- $(f + g)^*(x) = f^*(x) + g^*(x)$.
- $(f - g)^*(x) = f^*(x) - g^*(x)$.
- $(f \cdot g)^*(x) = f^*(x) \cdot g^*(x)$.
- $(f/g)^*(x) = f^*(x)/g^*(x)$, provided $g(x)$ and $g^*(x)$ are non-zero.

Show that these rules are contradictory. Give an example of a function for which you can get two different d'oh–rivatives, depending on how you apply the rules.

4. (10 points) Consider the function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

a. Compute the derivative of f. You will need to consider the cases $x \neq 0$ and $x = 0$ separately.

b. Show that $f'(x)$ is not continuous at $x = 0$.

5. (10 points) Prove, by induction, that $\frac{d}{dx}x^n = nx^{(n-1)}$, for any positive integer, n. Note, I gave a different, more difficult, proof of this fact in class. You now have more tools at your disposal that I did at the time.

6. (10 points) Draw the graph of the function $g(x) = (x^2 - 1)^3$ as accurately as possible. Label all maxima, minima, and inflection points.

7. (10 points) Prove that $\lim_{x \to 0} \frac{1}{x}$ does not exist. Hint: show that for any real number, L, it is impossible to respond to the challenge neighborhood $(L-1, L+1)$ and that, therefore, $\lim_{x \to 0} \frac{1}{x} \neq L$.

8. (10 points) a. Prove that if $f(x)$ is differentiable on $[a, b]$, then $\int_a^b f(x)\,dx$ exists.

b. Show that the converse is false; that is, show that a function which is integrable on $[a, b]$ does not have to be differentiable on all of $[a, b]$.

9. (10 points) Let $f(x) = x^2$. $f(x)$ is increasing on the interval $[2, 4]$, so there are *good* approximating step functions, $\sigma_n(x)$ and $\tau_n(x)$. Compute $\int_2^4 \sigma_4(x)\,dx$ and $\int_2^4 \tau_4(x)\,dx$.

10. (10 points) Prove that $1 + 1 = 2$.

More precisely: the axioms only assume that there are two real numbers, 0 and 1. Prove that $1+1 \neq 0$ and $1+1 \neq 1$. Therefore, $1+1$ must be a new real number, which we will call "2"

For the purpose of this problem, be extremely pedantic. Do not skip any steps, even those that you consider to be "trivial". You are free to use any theorems that we proved in class, but you must list all the theorems that you use at the bottom of the page.

List all theorems that you used, below.

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