MAT 141
ASSIGNMENT 2

DUE SEPT 13, 2005

Please write proofs (or at least some reasoning if you can't give a full proof), not just answers! So if a problem asks whether some set has a maximum, do not just write "No", write "No, it doesn't because..."

As usual, you are allowed all the theorems stated in the appropriate chapters of the book (whether or not we have discussed them in class).

Notation:

$\mathbb{Z}$ – set of integer numbers

$\mathbb{P}$ – set of positive integer numbers

$\mathbb{Q}$ – set of rational numbers

(1) Section I 3.12, problem 4

(2) Section I 3.12, problem 6 [Hint: consider first the case when $y - x > 1$, and show that in this case, there exists an integer r between them]

(3) (a) Show (without using logarithms) that the set $\{2, 4, 8 \dots\} = \{2^n \mid n \in \mathbb{P}\}$ is unbounded above. [Hint: can you show that $2^n > n$?]

(b) More generally, let $a_1, a_2, \dots$ be an increasing sequence of integer numbers: $a_1 < a_2 < a_3 < \dots$. Show that the set $\{a_1, a_2, \dots\}$ is unbounded above

(4) For each of the following sets, determine whether it is bounded above/below; whether it has a maximum/minimum (if so, find them); whether it has supremum/infinum (if so, find them)

(a) $\mathbb{Q}$

(b) $\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \dots\} = \{\sqrt{n} \mid n \in \mathbb{P}\}$

(c) $\left\{\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \dots\right\} = \left\{\frac{2^n - 1}{2^n} \mid n \in \mathbb{P}\right\}$

(d) $\left\{-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \dots\right\} = \left\{(-1)^n \frac{1}{n} \mid n \in \mathbb{P}\right\}$