MAT131 Fall 2010 Final Exam

Name: _____ SB ID number: _____

Please circle the number of your recitation.

1. TuTh 12:50 – SB Union Gianniotis

3. TuTh 8:20 – SB Union Lee

4. WF 11:45 – Lgt Engr Lab Boyd

5. MF 2:20 – Library Kim

6. MW 10:40 – Physics Medina

7. MW 6:50 – Physics Atyam

8. MW 3:50 – Old Chem Medina

9. TuTh 5:20 – SB Union Gianniotis

11. TuTh 3:50 – SB Union Lee

12. TuTh 8:20 – SB Union Wroten

13. MF 12:50 – Physics Kim

********** **DO NOT WRITE BELOW THIS LINE.** **********

Problem 1: _____ /30	**Problem 4**: _____ /25	**Problem 6**: _____ /25
Problem 2: _____ /40	**Problem 5**: _____ /20	**Problem 7**: _____ /20
Problem 3: _____ /40		**TOTAL**: _____ /000

Instructions: The exam is closed book, closed notes, calculators are not allowed, and all cell phones and other electronic devices must be turned off for the duration of the exam. You will have approximately 150 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please **show all work**, unless instructed otherwise. Partial credit will be given only for work shown. You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., then **please raise your hand**.

Name: _____ **Problem 1**: _____ /30

Problem 1(30 points) In each of the following cases, compute the limit. **Show all your work, and put a box around each answer.**

(a)(5 points)

$$\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)}$$

(b)(10 points)

$$\lim_{x\to 0} \frac{1-\cos(x)}{x^2}$$

Name: _____ **Problem 1, continued**

(c)(5 points)

$$\lim_{x \to 1} \frac{x^3 - x^{-3}}{x^2 - x^{-2}}$$

(d)(10 points)

$$\lim_{x \to 1} x^{1/(x-1)}.$$

Name: _____ **Problem 2**: _____ /40

Problem 2(40 points) For the following function $y = f(x)$, do each of the following.

$$y = \frac{6x^2 - 8}{x^3} = 6x^{-1} - 8x^{-3}$$

Show all your work.

(a)(4 points) Compute both $f'(x)$ and $f''(x)$ for $f(x) = 6x^{-1} - 8x^{-3}$. **Write each answer in the form $ax^{-m} + bx^{-n}$ where a, b, m, and n are whole numbers, and box each answer.**

(b)(1 points) Circle the answer below for the type of symmetry of $f(x)$.

Even **Odd** **Neither**

(c)(2 points) Find each vertical asymptote. **Write the equation of each vertical asymptote in the form $x = a$, and put a box around your final answer.**

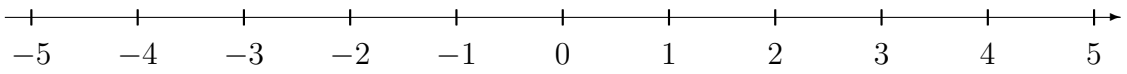
(d)(3 points) Find each horizontal asymptote. **Write the equation of each horizontal asymptote in the form $y = b$, and put a box around your final answer.**

(e)(2 points) Write the derivative of $6x^{-1} - 8x^{-3}$ in the form $g(x)/x^4$ where $g(x)$ is a polynomial. **Put a box around your final answer.**

Name: _____ **Problem 2, continued**

(f)(3 points) Find the x-coordinate of every critical point of $f(x)$. **Box your answer.**

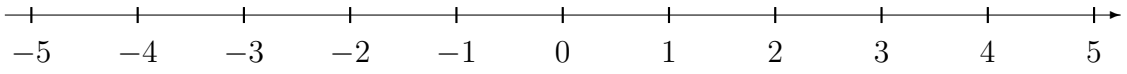
(g)(5 points) On the number line below, label the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.



(h)(2 points) Write the second derivative of $6x^{-1} - 8x^{-3}$ in the form $h(x)/x^5$ where $h(x)$ is a polynomial. **Put a box around your final answer.**

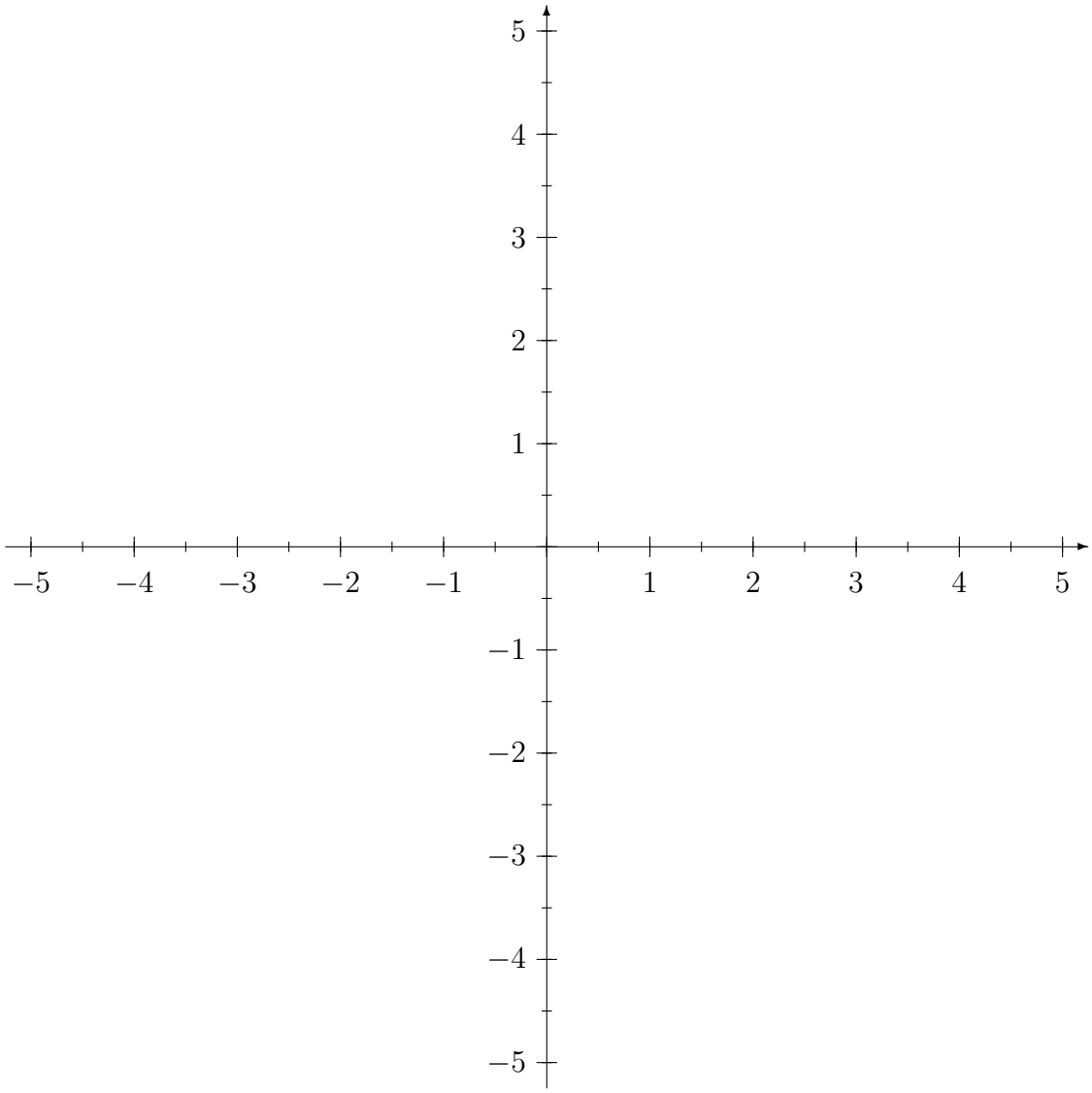
(i)(3 points) Find the x-coordinate of every inflection point of $f(x)$. **Box your answer.**

(j)(5 points) On the number line below, label the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down. (The approximation $\sqrt{2} \cong 1.4$ might be helpful.)



Name: _____ **Problem 2, continued**

(k)(10 points) On the grid below, sketch the graph of $y = 6x^{-1} - 8x^{-3}$. Carefully label every vertical asymptote, every horizontal asymptote, every local maximum, every local minimum, and every inflection point.



Name: _____ **Problem 3**: _____ /40

Problem 3(40 points) Compute each of the following integrals. **Show all work, and put a box around each answer.**

(a)(5 points)

$$\int_1^{\sqrt{2}} (x^3 - x)dx.$$

Write your answer as a fraction of whole numbers.

(b)(5 points)

$$\int_{-\pi/6}^{+\pi/6} 2\cos(x)dx.$$

Write your answer as a fraction of whole numbers.

(c)(5 points)

$$\int \frac{1}{1+t^2}dt.$$

(d)(5 points)

$$\int_{-e^5}^{-e^2} \frac{1}{x} dx.$$

Write your answer as a fraction of whole numbers.

(e)(10 points)

$$\int_{\pi/4}^{\pi/3} \frac{\sec^2(x)}{\tan(x)} dx.$$

Write your final answer without any trigonometric functions, square roots or exponents.

(f)(10 points)

$$\int \frac{x^5 + x^2}{(x^3+1)^2} dx.$$

Name: _____ **Problem 4**: _____ /25

Problem 4(25 points) A tall metal box has squares of edge length x for the bottom and top. The sides have height y. The total volume is 1000 cubic feet.

The cost of production is proportional to the total length of those edges of the box which must be welded: 3 of the 4 edges of the top, 3 of the 4 edges for the bottom, and 1 of the 4 edges for the side. What choice of x and y minimizes the total length of the edges to be welded? **Show all work and put a box around your answer.**

Name: _____ **Problem 5**: _____ /20

Problem 5(20 points) Consider the function $f(x) = 1/(x^2 + 1)$. Use the limit definition of the derivative to compute $f'(x)$.

Name: _____ **Problem 6**: _____ /25

Problem 6(25 points) A point moves on the ellipse with equation

$$3x^2 - 2xy + 3y^2 = 12.$$

When the coordinates of the point are $(0, 2)$, dx/dt equals 6. At that moment, what is the rate of change of the distance between the point and the origin? **Show all work, write your answer as a fraction, and put a box around your answer.**

Name: _____ **Problem 7**: _____ /20

Problem 7(20 points) Consider the limit,

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{i}{n} \left(1 - \frac{i^2}{n^2}\right).$$

(a)(10 points) Find a definite integral whose limit of approximating sums is the expression above.

$$\int_0^1 f(x)dx = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{i}{n} \left(1 - \frac{i^2}{n^2}\right).$$

Show all work, and put a box around your final answer.

(b)(10 points) Compute the limit by evaluating the integral. **Write your answer as a fraction, show all work, and box your final answer.**

SCRATCH PAGE