MAT 123 Introduction to Calculus — Jason Starr
Core Competency Exam A — Spring 2014
In-lecture Wednesday / Thursday, Week of 2/16-2/20

MAT 123 Practice for Core Competency Exam A With Solutions

Remark. If you are comfortable with all of the following problems, you will be well prepared for Core Competency Exam A. For the Core Competency Exams, passing will be 85% or better.

Exam Policies. You must show up on time for all exams. Please bring your student ID card: ID cards may be checked, and students may be asked to sign a picture sheet when turning in exams. Other policies for exams will be announced at the beginning of the exam.

If you have a university-approved reason for taking an exam at a time different than the scheduled exam (because of a religious observance, a student-athlete event, etc.), please contact your instructor as soon as possible. Similarly, if you have a documented medical emergency which prevents you from showing up for an exam, again contact your instructor as soon as possible.

For excused absences from a Core Competency Exam, since there will be multiple attempts for such an exam, usually the student will simply be asked to pass one of the later attempts. In exceptional circumstances, a make-up exam may be scheduled for the missed exam.

All exams are closed notes and closed book. Once the exam has begun, having notes or books on the desk or in view will be considered cheating and will be referred to the Academic Judiciary.

For all exams, you must bring your Stony Brook ID. The IDs may be checked against picture sheets.

It is not permitted to use cell phones, calculators, laptops, MP3 players, Blackberries or other such electronic devices at any time during exams. If you use a hearing aid or other such device, you should make your instructor aware of this before the exam begins. You must turn off your cell phone, etc., prior to the beginning of the exam. If you need to leave the exam room for any reason before the end of the exam, it is still not permitted to use such devices. Once the exam has begun, use of such devices or having such devices in view will be considered cheating and will be referred to the Academic Judiciary. Similarly, once the exam has begun any communication with a person other than the instructor or proctor will be considered cheating and will be referred to the Academic Judiciary.

Review Topics.
The following are the core skills for the first part of the course.

(1) Recognize the domain of a sketched function. Recognize where a sketched function has positive y-values and where it has negative y-values.

(2) Understand the meaning of the absolute value function. Be able to translate between equations using absolute value functions, and equivalent formulations using piecewise-defined functions.

(3) Understand basic transformations of graphs, particularly reflections.

(4) Be able to add, subtract, multiply and divide functions, including polynomial functions and fractions of polynomial functions. Be able to simplify your answer. Be able to compose two given functions.

(5) Be able to solve a quadratic equation. Use this to find where a parabola is above or below the y-axis. Use this to find the intersection points of a line and a conic section.

(6) Be able to factor polynomials in simple cases. Be able to check a given general factorization.

(7) Find the equation of a given line in slope-intercept form, in slope-point form, and in point-point form. Conversely, find the slope and intercepts of a line with a given equation.

(8) Check whether two given lines are parallel, respectively perpendicular. For a given line and a given point, find the line containing the point and parallel, resp. perpendicular, to the given line.

(9) Understand qualitative features of a parabola: its vertex, whether it opens up or down, and its x-intercepts (assuming these exist). Be able to determine these features for a given equation of a parabola.

Practice Problems.

(1) In each of the following cases, determine whether the given equation: (A) has no solutions, (B) has a unique solution, (C) has one positive and one negative solution, (D) has two positive solutions, or (E) has two negative solutions.

(i) $|2x+2| = 0$, (ii) $|2x+1| = 2$, (iii) $|2x-3| = 2$, (iv) $|2x+3| = 2$, (v) $|2x+2|+1 = 0$.

(2) The following inequality,

$$|3x+5| \leq 2,$$

defines the following subset of real numbers,

(a) $(-\infty, -1]$, (b) $(-7/3, -1)$, (c) $[-7/3, -5/3) \cup (5/3, -1]$, (d) $[-7/3, -1]$, (e) $[0, \infty)$.

(3) For a pair (x, y) that satisfies

$$|2x+1| = |2y+1|,$$

y must equal,

$$\text{(a) } x, \quad \text{(b) } x \text{ or } -x, \quad \text{(c) } -x-1, \quad \text{(d) } x \text{ or } -x-1, \quad \text{(e) } -x \text{ or } -x-1.$$

(4) For each of the following identities, say whether or not it is (always) true or (sometimes) false.

$$\text{(i) } |x-2| = |2-x|, \quad \text{(ii) } |3x| = 3|x|, \quad \text{(iii) } |-2x| = -2|x|.$$

(5) For the graph of a function $y = f(x)$, match each of the following functions,

$$\text{(i) } f(-x), \quad \text{(ii) } -f(x), \quad \text{(iii) } -f(-x).$$

to the corresponding transformation of the original graph,

(a) reflection through the origin , (b) reflection through the x-axis , (c) reflection through the y-axis.

(6) For the function $f(x) = 1/x^2$, $f(f(2))$ equals

$$\text{(i) } 1/4, \quad \text{(ii) } 16, \quad \text{(iii) } 4, \quad \text{(iv) } 1/16.$$

(7) For the function $f(u) = u^2 - 1$ and the function $g(x) = 2x + 1$, $f(g(x))$ equals,

$$\text{(i) } 4x(x+1), \quad \text{(ii) } 2(x^2-1)+2, \quad \text{(iii) } 2x^2, \quad \text{(iv) } (2x+2)^2 - 1.$$

(8) A given line has x-intercept $(-3, 0)$ and y-intercept $(0, 1)$. For the point on the line with x-coordinate 2, the y-coordinate equals

$$\text{(i) } 3, \quad \text{(ii) } -3, \quad \text{(iii) } \frac{2}{3} - 1, \quad \text{(iv) } \frac{5}{3}.$$

(9) The equation of the line with slope -5 and containing the point $(2, 0)$ is

$$\text{(i) } y = -5x + 2, \quad \text{(ii) } y = -5x + 10, \quad \text{(iii) } (y-0) = (x-2)/(-5), \quad \text{(iv) } y = 5(x-2).$$

(10) If two points on a given line are $(-2, 2)$ and $(-3, 0)$, then the equation of the line is

$$\text{(i) } (y-1) = \frac{-2-(-3)}{2-0}(x-(-3)), \quad \text{(ii) } , y = 2x+6 \quad \text{(iii) } , (y-2) = 2(x+(-2)) \quad \text{(iv) } y + x = 0.$$

(11) For the following line,

$$3y + 2x + 4 = 0,$$

the y-intercept is the point

$$\text{(i) } (0, -4/3), \quad \text{(ii) } (-2, 0), \quad \text{(iii) } (2, 0), \quad \text{(iv) } (0, 4/3).$$

(12) The parabola $y = x^2 + x$ lies above the x-axis precisely for those x-values in the set,

$$\text{(i) } (-1, 0), \quad \text{(ii) } (-\infty, -1) \cup (0, \infty), \quad \text{(iii) } (-\infty, 0), \quad \text{(iv) } (-1, \infty).$$

(13) For the following line,

$$3y + 2x + 4 = 0,$$

the line perpendicular to this line and containing the point $(1, 1)$ has the equation

$$\text{(i) } 2y + 3x = 4, \quad \text{(ii) } y - 1 = \frac{-2}{3}(x - 1), \quad \text{(iii) } y = \frac{-2}{3}x + \frac{5}{3}, \quad \text{(iv) } y = \frac{3}{2}x - \frac{1}{2}.$$

(14) The parabola with equation

$$y = 2x(x + 2)$$

has

(i) vertex at $(1, 6)$ and opens up,

(ii) vertex at $(1, 6)$ and opens down,

(iii) vertex at $(-1, -2)$ and opens up,

(iv) vertex at $(-1, -2)$ and opens down.

(15) The distance from the line with equation

$$y = 2x + 1$$

to the point $(1, 1)$ equals

$$\text{(i) } 2\sqrt{5}/5, \quad \text{(ii) } \sqrt{18}/\sqrt{5}, \quad \text{(iii) } \sqrt{\left(\frac{7}{5} - 1\right)^2 + \left(\frac{1}{5}\right)^2}.$$

Solutions.
Problem 1 (i) B, (ii) C, (iii) D, (iv) E, (v) A.
Problem 2 d
Problem 3 d
Problem 4 (i) T, (ii) T, (iii) F.
Problem 5 (i) c, (ii) b, (iii) a.
Problem 6 (ii).
Problem 7 (i).
Problem 8 (iv).
Problem 9 (ii).
Problem 10 (ii).
Problem 11 (i).
Problem 12 (ii).
Problem 13 (iv).
Problem 14 (iii).
Problem 15 (i).