MAT 544 Problem Set 2

Homework Policy. Write up solutions of the required problems. Read and attempt the extra problems, but do not write up those solutions for grading. Many of these problems are taken from or inspired by problems in the textbook.

Each student is encouraged to work with other students, but submitted problem sets must be in the student's own words and based on the student's own understanding. It is against university policy to copy answers from other students or from any other resource.

Required Problems.

Problem 1. Write a careful solution to Problem 1 on the diagnostic exam: every group of order 231 is isomorphic to either $\mathbb{Z}/231\mathbb{Z}$ or to $\Gamma \times (\mathbb{Z}/11\mathbb{Z})$ where $\Gamma \subset \mathbf{SL}_2(\mathbb{Z}/7\mathbb{Z})$ is the subgroup of upper triangular matrices whose diagonal entries are cube roots of unity in $\mathbb{Z}/7\mathbb{Z}$.

Problem 2. Write a careful solution to Problem 2 on the diagnostic exam. For every characteristic 0 field F that contains a primitive cube root of unity, there is a natural bijection between the set of F-isomorphism classes of Galois extension E/F with Galois group isomorphic to $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$ and the subgroups of $F^\times/(F^\times)^3$ isomorphic to $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$ that associates to E/F the subgroup $((E^\times)^3 \cap F^\times)/(F^\times)^3$ of $F^\times/(F^\times)^3$.

Problem 3.(Atiyah and MacDonald, p. 32, 2.3) For every local ring A, for nonzero finitely generated A-modules M and N, prove that also $M \otimes_A N$ is nonzero.

Problem 4.(Atiyah and MacDonald, p. 32, 2.8) For A-flat A-modules M and N, prove that also $M \otimes_A N$ is A-flat. For every flat A-algebra B, and for every B-flat B-module P, prove that also P is A-flat.

Problem 5.(Atiyah and MacDonald, p. 32, 2.9) For a short exact sequence of A-modules, if the first and third modules are finitely generated, then so is the middle module. Then use the Snake Lemma to prove that if the first and third modules are finitely presented, then so is the middle module.

Problem 6.(Atiyah and MacDonald, p. 32, 2.19) Prove that for every filtering partially ordered set I, prove exactness of the functor from the category of I-compatible systems of A-modules to the category of A-modules that sends each I-system to its colimit.

Problem 7.(Atiyah and MacDonald, p. 32, 2.20) Prove that tensor product commutes with colimits.