

Your Majesty, Your Excellencies, ladies and gentlemen:

I'm very happy to be in this beautiful country again, and very much honored by this award.

The field of mathematics is a marvelous mosaic built up out of contributions by people from many different cultures, speaking many different languages, and stretching back over many hundreds of years. From the beginning mathematics has had a dual nature, partly abstract and self contained, but intimately concerned with understanding of the physical world around us. Much important mathematics was first inspired by real world problems; and mathematics has often contributed in totally unexpected ways. No one could have guessed that Riemann's study of curvature would form the basis for Einstein's theory of gravity; or that Hilbert's theory of infinite dimensional vector spaces would provide the foundations for quantum mechanics. The British mathematician G. H. Hardy proudly bragged that his work in number theory would never be sullied by applications. He would have been horrified to learn that it is now the basis for methods in cryptography which are fundamentally important in commercial applications and also in military applications.

The connections between mathematics and other sciences work in both directions. Claude Shannon's work on communication theory was inspired by the work of physicists on statistical mechanics, and now has important applications not only in computer science but also in the mathematical theory of dynamical systems. The mathematical theory of Riemann surfaces is now very important to mathematical physicists. Conversely, tools developed by mathematical physicists play a very important role in topology.

I have been very lucky to have been able to enjoy this magnificent mosaic of mathematics for more than sixty years, and to make some contributions to it. But of course the contributions of any one person must depend in a very essential way on the cumulative contributions of older generations of mathematicians. The work of Niels Henrik Abel provides a necessary background for a great deal of present day mathematics. The groups studied by Sophus Lie are important in many branches of mathematics.

We learn not only from our mathematical ancestors, but also to a great extent from our contemporaries. I have personally benefited from the work of a number of the previous Abel Prize winners. The thesis of Jean-Pierre Serre provided a foundation for nearly all subsequent work on homotopy groups. His beautiful *Cours d'Arithmétique* taught me about the quadratic forms which play an important role in understanding the topology of manifolds. In fact, this study of quadratic forms was so addictive that I spent some years studying problems in algebra for their own sake. Michael Atiyah's work on K-theory provided the inspiration for my own work on algebraic K-theory; while John Tate helped me to understand the relationship between algebraic K-theory, quadratic forms and Galois cohomology.

One great advantage of the long mathematical life which I have enjoyed is that it has enabled me to see amazing progress by others on problems which I had helped to formulate. Thus the work on quadratic forms which I just mentioned led to conjectures which were later verified in very deep work by Vladimir Voevodsky. Similarly, Misha Gromov's work on the growth of finitely generated groups went far beyond anything which I had been able to achieve.

In conclusion, let me say that I am very grateful to the Abel Committee for this award. I want to thank all of you here today for helping me to celebrate it.