Interpolation by Smooth Functions

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ABSTRACT

Fix a finite subset $E \subset \mathbb{R}^n$, and an integer $m \geq 1$. Given data $f : E \to \mathbb{R}$, how small can one make the m-th derivatives of an interpolant of f defined on $\mathbb{R}^n$? Are there efficient algorithms to compute an interpolant whose m-th derivatives are nearly minimal? What about keeping the m-th derivatives nearly minimal *on average*?

Initial results [1, 3, 4] provided near-linear time algorithms that interpolate by functions with uniformly controlled derivatives. In recent work [2], new methods were developed to treat both the uniform and average problems. I believe that these new methods can be implemented as near-linear time algorithms, but the most obvious way is inefficient.

Let me describe this new method for interpolating data by functions with uniformly small second derivatives in two-dimensions (i.e., $m = n = 2$).

For some subsets the problem is easy: If E consists of three non-collinear points in the plane, or at most two points in the plane, then any data $f : E \to \mathbb{R}$ can be interpolated with an affine function.

There are also subsets having intermediate difficulty: If E lies on a nice smooth curve in the plane, then the problem can be reduced to a one-dimensional interpolation problem that is readily solved.

It turns out that solving one-dimensional problems can help to solve the generic problem. The approach proceeds by partitioning the given (arbitrary) subset E into pieces E_ν, each lying on a smooth curve. Thanks to the previous paragraph, we can solve for local interpolants F_ν that match the data on each piece, and patch them together with a partition of unity to produce the global interpolant F. In order to maintain good control on the second derivatives of F, we must carefully choose the F_ν to be mutually consistent. The manner by which this choice is made is the novel part of this approach.

For higher values of m and n, the method is similar as above, but with more layers of complexity between easy problems and generic problems.

BIO

I did my undergrad at Florida Atlantic University, studied under Charles Fefferman for my Ph.D. at Princeton Univerity, and am currently a postdoc at the Courant Institute working on metric geometry and extension/interpolation problems.

1. REFERENCES

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