Math53: Ordinary Differential Equations
Winter 2004

Final Exam Information

Monday, 3/15, 7-10p.m.

Building 370-Lecture Hall 370 (*ground floor*)

General Information

This will be a closed-book, closed-notes exam. No calculators will be allowed. The final exam will cover the entire course. There will be ten problems. In my view, it is about three times as long as the fifty-minute, four-problem, 100-point Midterm I. It will be graded out of 300 points. *In contrast* to the two midterms, there will be no space left between parts of a problem, and you should write your answer after the question or on a separate sheet of paper. In one case, you will be asked to use a full page for your answer. There will also be problems with no parts at all, e.g. you might be asked to solve an IVP and it will be up to you to choose the method.

End-of-Quarter Office Hours

I will have office hours today 4-6, Friday 10-12 and 4-6, Sunday 10-12 and after the review session (at least until 6pm), Monday 10-12 and 3-5, in 383B. The course assistant, Isidora Milin, will have office hours on Thursday 3:15-5:15 and Monday 12-2, in 380S. The last SUMO tutoring session is tonight 6-10 in 381T. *Please do come if you have any questions.*

Review Session

There will be a review session on Sunday starting at 3pm in 383N, on the third floor. Part I of the review session will cover Chapters 1-5. Part II will cover Chapters 6-10; it will start after all questions from the first five chapters are answered, but not before 4p.m. During the first 10 minutes or so of each part, I will summarize the key points. After that I will only answer questions, whether specific or general (such as working out an example of something). After the review session, I will hold office hours, at least until 6p.m., to answer any remaining questions. This review session is completely optional; feel free to skip either of its parts if you like.

More about the Final Exam

(1) There will be one application-style problem, motion or mixing. However, there will be no numbers to plug in; instead, you'll need to derive a formula.
(2) There will be one problem involving the first-second and second-order numerical methods. You will need to carry out the computations using simple fractions, i.e. p/q, like in the lecture.
(3) There will be a problem involving either a high-order ODE with easy-to-find characteristic roots or a 3×3 linear system with easy-to-find eigenvectors.
(4) There will be a variation-of-parameters problem, though this term will not be used.
(5) As there was no phase-plane portrait with repeated eigenvalues on the second midterm, you should certainly expect one on the final, either with nonzero or zero double eigenvalue. On the other hand, any of the five generic cases could easily appear as part of a nonlinear phase-plane portrait.
(6) You do not have to memorize the real form of the general solution for a planar system with complex eigenvalues, if you can solve an IVP using the complex form and then simplify it to a real expression, as done in the PS5 Solutions (see 6.1:18).
(7) You will not have to use the Laplace Transform for any problem, but you could decide to do so. It is far faster to solve some IVPs using the Laplace Transforms. The two LT tables from the second midterm will also appear on the final exam.

Suggested Preparation

Make sure you can do the final exam practice problems, the practice and the actual midterms, all problem set exercises from the textbook, and perhaps some other related problems from the textbook. You may also want to look over the six lecture summaries and the solutions to the six problem sets and to the two midterms, even if you know how to do every question. I am planning to post solutions to the final exam practice problems, perhaps as part of a course summary, on the course website on Sunday morning.

During the Final Exam

If possible, try to confirm your answers by direct checks. For example, if you find a solution to an IVP, check that it satisfies the initial condition and the ODE. If you find an eigenvector for a matrix, check that the matrix times the vector is the expected multiple of the vector. The latter is likely to take only seconds in the two-dimensional case.

Important Concepts

(1) *Conceptual Things:* solutions to IVPs; implicitly defined solutions; direction fields; applications of Existence and Uniqueness Theorems; first-order autonomous equations (equilibrium points, stability, phase line, sketching solution curves, limiting behavior); autonomous systems of first-order equations (equilibrium points, stability, limit cycles); component plots and phase-plane portraits; planar autonomous systems of first-order ODEs; high-order equations and systems of first-order equation; linear independence; structure of solutions of linear ODEs. Examples: 2.1:8,17-20; 2.7:1-6,25-32; 2.9:15-28; 4.1:5-12; 8.1:7-16; 8.2:13-21; 9.6:1-14.

(2) *Solving First-Order ODEs:* find solutions of linear, separable, and exact ODEs; check for exactness; determine correct constant, interval of existence, and/or square root for an IVP; sketch solution curves. Examples: 2.2:1-18, 2.4:1-8,13-21, 2.6:9-21,35-40.

(3) *Applications of First-Order ODEs:* motion; mixing. Examples: 2.3:1-6; 2.5:1-7.

(4) *Solving Second-Order Linear ODEs with Constant Coefficients:* characteristic roots and the general solution of a homogeneous ODE; find a particular solution and the general solution of an inhomogeneous ODE; the method of undetermined coefficients. Examples: 4.3:1-28; 4.5:1-47 (but choose your own method).

(5) *Variation of Parameters:* given a nonzero solution of a linear homogeneous ODE, find another linearly independent solution; given two linearly independent solutions of the corresponding homogeneous ODE, find a particular solution of an inhomogeneous ODE. Examples: 4.1:14-18; 4.6:13,14.

(6) *Laplace Transform:* use LT to solve IVPs; convolution. Examples: 5.4:1-36; 5.7:17-22,26-31.

(7) *Linear Algebra:* nullspace of a matrix; basis for the nullspace; determinant; find the inverse of a matrix; find eigenvalues, eigenvectors, and generalized eigenvectors; find the exponential of a matrix; find the product of the exponential of a matrix and of a vector. Examples: 7.4:7-22; 7.5:1-40; 7.6:7-10,13-15,22-45,50-54; 9.1:1-12; 9.5:1-16,25-28.

(8) *Planar Systems of Linear ODEs with Constant Coefficients:* find the general solution to planar systems, homogeneous and inhomogeneous; solve initial value problems; sketch phase-plane portraits for planar homogeneous systems. Examples: 9.2:1-12, 23-64 + phase-plane plots; 9.8:1-24.

(9) *Higher-Order Equations and Higher-Dimensional System:* find the general solution of a linear homogeneous ODE with constant coefficients; find the general solution of a system of first-order linear homogeneous ODEs with constant coefficients. Examples: 9.4:7-11,21-26; 9.7:14-33.

(10) *Qualitative Analysis of Nonlinear Autonomous Systems of ODEs:* find equilibrium points; use the two jacobian tests to determine their type; nullclines and phase-plane sketches; limit cycles; conserved quantities. Examples: 10.1:1-8,19a,20; 10.2:1-10; 10.3:1-4,9-12,13-16; 10.4:1-7;10.5:1-10, +phase-plane sketches.

(11) *Numerical Methods:* applications of first-order and second-order methods. Examples: 6.1:1-5,18-21; 6.2:1-5.