

Math53: Ordinary Differential Equations Winter 2004

Midterm I Practice Tests

Practice Test A

Problem 1

- (a) Find the general solution to the ODE

$$yy' = -2t - 2ty^2.$$

- (b) Find an explicit solution, including the interval of existence, to the initial value problem

$$yy' = -2t - 2ty^2, \quad y(0) = 1.$$

Problem 2

- (a) Show that the ODE

$$1 - y \sin t + (\cos t)y' = 0$$

is exact and solve it for $y=y(t)$, implicitly or explicitly.

- (b) Find an explicit solution, including the interval of existence, to the initial value problem

$$1 - y \sin t + (\cos t)y' = 0, \quad y(0) = 1.$$

Problem 3

- (a) Find the general solution of the ODE $y'' - y = 0$.
(b) Find a solution of the ODE $y'' - y = 2t$.
(c) Find a solution of the ODE $y'' - y = 2e^{-t}$.
(d) Find the general solution of the ODE $y'' - y = 2t - 4e^{-t}$.

Problem 4

Consider the initial value problem

$$y' = \sqrt{|y|}/|t|, \quad y(t_0) = y_0.$$

For what initial conditions (t_0, y_0) does the existence and uniqueness theorem for first-order ODEs

- (a) guarantee a solution for this IVP?
(b) guarantee a unique solution for this IVP?
Specify all such conditions.

Practice Test B

Problem 1

- (a) Find the general solution to the ODE

$$ty' = \sin t - 2y.$$

Sketch at least three solution curves.

- (b) Find the solution, including the interval of existence, to the initial value problem

$$ty' = \sin t - 2y, \quad y(\pi/2) = 0.$$

Problem 2

- (a) Show that the substitution $y = tv$ reduces the ODE

$$t + y + (y - t)y' = 0 \quad \text{to} \quad tv' = \frac{1 + v^2}{1 - v}.$$

- (b) Find the general solution to the latter ODE.
(c) Find the solution, explicitly or implicitly, to the initial value problem

$$t + y + (y - t)y' = 0, \quad y(1) = 0.$$

Problem 3

- (a) Find the general solution of the ODE $y'' + 4y' = 0$.
(b) Find a solution of the ODE $y'' + 4y' = 20 \cos 2t$.
(c) Find the solution to the initial value problem

$$y'' + 4y' = 16 \cos 2t, \quad y(0) = 1, \quad y'(0) = 1.$$

Problem 4

- (a) Sketch the graph of the function

$$f(y) = (y + 3)^2(y + 1)(y - 3).$$

- (b) Find and sketch the equilibrium solutions of the ODE

$$y' = (y + 3)^2(y + 1)(y - 3).$$

- (c) On the same plot, sketch at least one solution curve of this ODE in each region of the ty -plane cut out by the graphs of the equilibrium solutions. Indicate the asymptotic behavior. Determine whether each of the equilibrium solutions is asymptotically stable or unstable. Draw the phase line.

Remarks: Problems A1, A2, A3, B1, B2, B3 are similar to 2.2:14, 2.6:10, 4.5:32, 2.4:13, 2.6:36, and 4.5:26; see PS1 and PS2 solutions. Problems A3 and B3 also make use of 4.5:30. For Problems A4 and B4, see the last section of *Unit 2 Summary*. The two practice tests are meant to indicate some problems that might appear on the midterm and its length, and *not* that the midterm problems would come from the problem sets.