Math53: Ordinary Differential Equations
Winter 2004

Midterm I Information

Wednesday, 1/28, 2:15-3:05p.m.

Last Name A-L: 380Y   *Last Name M-Z:* 380W

General Information

This will be a closed-book, closed-notes exam. No calculators will be allowed. Midterm I will cover Chapters 1-4 of the textbook, but the focus will be on Sections 1.2, 1.3, 2.1, 2.2, 2.4, 2.6, 2.7, 2.9, 4.1, 4.3, 4.5, and 4.6. I suggest you look in detail over these sections, the two lecture summaries handed out in class, and the solutions to the first two problem sets. Make sure you can do all problem set exercises from these twelve sections and perhaps some other related problems from the textbook. There will be no application-style problems, like 2.3:4, 2.5:4, or 3.1:12 on the midterm, but they could appear on the final exam.

Pre-Midterm I Office and Tutoring Hours

Over the next few days, I will have office hours Sunday 4-6, Monday 10-12, Tuesday 4-6, and Wednesday 10-12, in 383B. Most of these office hours are for this week only. As always, the course assistant, Isidora Milin, will have office hours Monday 12-1 and Wednesday 12-2 in 380S. The SUMO tutoring is on Monday 6-10 in 381T.

Background Material

You should be familiar with and know how to use the two FTCs, chain and product rules, integration-by-parts and change-of-variables formulas. You may also encounter integrals like

$$\int te^{rt}dt, \qquad \int \frac{1}{t(t+1)}dt, \qquad \int e^{rt}\cos\omega t\, dt.$$

The last integral is computed in calculus by two integrations by parts, by it is far easier to use Euler's formula. If you encounter an integral you cannot compute quickly, you may instead want to replace it by an antiderivative and continue on, but be as specific as possible about your choices. For example, if you cannot compute the middle integral above, you may want to write some like: for $t>0$, let $F(t)=\int_1^t \frac{ds}{s(s+1)}$.

Types of Problems to Expect

(1) *Direction Fields:* given first-order ODEs, sketch the corresponding direction fields; given several first-order ODEs and several direction fields, match each ODE with its direction field. Examples: 2.1:17-20.

(2)*Existence and Uniqueness Theorem:* given a number of IVPs, determine to which ones the existence and uniqueness theorem applies; describe long-term behavior of solutions; Examples: 2.7:1-6, 2.7:25-32.

(3) *Autonomous Equations:* find equilibrium points, determine their type, show phase line, sketch solutions curves, and describe their limiting behavior. Examples: 2.9:15-28.

(4) *Solutions of ODEs:* given several functions and several ODEs, match each function with the ODE it solves; check that a given implicitly defined function solves an ODE. Example: 2.1:8.

(5) *Solving First-Order ODEs:* find solutions of linear, separable, and exact ODEs; check for exactness; determine correct constant, interval of existence, and/or square root for an IVP; sketch solution curves. Examples: 2.2:1-18, 2.4:1-8,13-21, 2.6:9-21,35-40.

(6) *Solving Second-Order Linear ODEs with Constant Coefficients:* find the characteristic roots and the general solution of a homogeneous ODE; find a particular solution of an inhomogeneous ODE, using the method of undetermined coefficients of Section 4.5 or the integrating-factor approach of PS2-Problem 1; Wronskian and linear independence; structure of solutions of linear ODEs. Examples: 4.1:5-12, 4.3:1-28; 4.5:1-47 (but choose your own method).

(7) *Variation of Parameters:* given a nonzero solution of a linear homogeneous ODE, find another linearly independent solution; given two linearly independent solutions of the corresponding homogeneous ODE, find a particular solution of an inhomogeneous ODE. Examples: 4.1:14-18; 4.6:13,14.

(8) *Higher-Order Equations:* find the characteristic roots and the general solution of a linear homogeneous ODE with constant coefficients.

Remarks: (1) You will not be asked to use a specific method to solve a second-order linear nonhomogeneous equation. Thus, if you feel comfortable using the method of undetermined coefficients of Section 4.5, you do not need to memorize the main formula involved in the integrating factor approach for second-order ODEs. However, you should know how to construct the general solution of a linear homogeneous ODE with constant coefficients from the roots of its characteristic polynomial.
(2) You will not need to memorize Kirchhoff's laws and similar things for either the midterm or the final. However, for the final exam, you should know that the velocity and the acceleration are the first and second derivatives of the displacement and that the concentration of a salt in water is its mass divided by the volume.