Math53: Ordinary Differential Equations
Autumn 2004

Homework Assignment 2

Problem Set 2 is due by 2:15p.m. on Monday, 10/11, in MuddChem 101

Problem Set 2:

2.6: 10,14,26,36; 2.9: 20,26,28; 4.3: 4,10,14,26; 4.4: 17 (1st part only); Problem B (see next page)

Note: While the statement of Problem B looks long, most of it is actually a review.

Daily Assignments:

Please review complex numbers, pp181-184, before Thursday, 10/7

Date	*Read*	*Exercises*
10/4 M	2.9	2.9:20,26,28
10/5 T	2.6	2.6:10,14,26,36
10/6 W	4.3 (pp181-184)	
10/7 R	4.1,4.3	Problem B
10/8 F	4.3,4.4	4.3:4,10,14,26; 4.4:17 (1st part only)

General hint: Doing computations with complex exponentials is usually easier than with real trigonometric functions.

Problem B

Let p and q be two constants. Suppose λ_1 and λ_2 are the two roots of the characteristic polynomial

$$\lambda^2 + p\lambda + q = 0 \tag{1}$$

associated to the linear homogeneous second-order ODE

$$y'' + py' + qy = 0.$$

As stated in class,

$$\big(e^{(\lambda_1-\lambda_2)t}(e^{-\lambda_1 t}y)'\big)' = e^{-\lambda_2 t}(y'' + py' + qy). \tag{2}$$

Thus, every second-order linear ODE with constant coefficients,

$$y'' + py' + qy = f(t) \tag{3}$$

can be solved in four steps:
Step 1: find the roots of the associated characteristic polynomial (1);
Step 2: multiply both sides of (3) by $e^{-\lambda_2 t}$;
Step 3: use (2) to compress LHS of the resulting expression and to obtain

$$\big(e^{(\lambda_1-\lambda_2)t}(e^{-\lambda_1 t}y)'\big)' = e^{-\lambda_2 t}f(t); \tag{4}$$

Step 4: solve (4) for y by integrating twice.

This approach mimics the *integrating factor method* for solving linear first-order ODEs, though it works *only* for constant p and q. Its advantage over the methods described in Sections 4.3 and 4.5 of the text is that
(1) it works the same way whether or not λ_1 and λ_2 are distinct;
(2) it works the same way no matter what f looks like.

Use the above second-order integrating factor method to find the *real* (not complex) general solutions of
(a) $y'' + 4y = 4\cos 2t$;
(b) $y'' + 5y' + 4y = t \cdot e^{-t}$.