MAT 648: Mirror Symmetry for Gromov-Witten Invariants
Fall 2014

Course Instructor

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Course Website

All updates, including schedule and references, will be posted on the course website,

`http://math.sunysb.edu/∼azinger/mat648`.

Please visit this website regularly.

Prerequisites

Passing the comps is required. You also should have some understanding of complex geometry (MAT 545) and algebraic topology (MAT 539). Feel free to contact me with any questions.

Grading

Your grade will be based on class participation, in all possible forms. The notes include some exercises, which you are encouraged to do; do not hand them, but I'd be happy to discuss them.

Course Description

This course is an introduction to computations in Gromov-Witten theory which use the Atiyah-Bott Equivariant Localization Theorem. We will apply it to obtain (classical) mirror symmetry statements for the genus 0 Gromov-Witten invariants of certain hypersurfaces and complete intersections. While these particular statements were first obtained 15-20 years ago, similar methods continue to be used today. The course will tentatively have the four parts listed on the next page. If time permits, I will also discuss stable quotients invariants (also known as quasi-map invariants).

Part I. *Equivariant Cohomology and the Equivariant Localization Theorem*

This part is pretty much all algebraic topology with a little bit of Lie algebras. I will give a self-contained introduction to equivariant cohomology with lots of examples and then prove the Atiyah-Bott Equivariant Localization Theorem as in the original paper, with all the details. It is a powerful tool for computing Euler classes of vector bundles over compact manifolds.

[AB] M. Atiyah and R. Bott, *The moment map and equivariant cohomology*, Topology 23 (1984), 1–28.

[Z] A. Zinger, *Notes on Mirror Symmetry*, Chapter 1.

Part II. *Stable maps.*

This will be an overview of the theory of stable (pseudo-) holomorphic maps. They are used to define Gromov-Witten invariants, which are certain counts of complex curves that have been the basis for many interactions between symplectic topology, algebraic geometry, and string theory.

[MS] D. McDuff and D. Salamon, *J-Holomorphic Curves and Symplectic Topology*, 2nd Ed., AMS 2012, Chapters 5-7.

[MirSym] K. Hori, S. Katz, A. Klemm, R. Pandharipande, R. Thomas, C. Vafa, R. Vakil, and E. Zaslow, Mirror Symmetry, Clay Math. Inst., AMS 2003, Chapters 21-24.

Part III. *Genus 0 mirror symmetry for projective hypersurfaces.*

I will overview the 1991 mirror symmetry prediction which relates the genus 0 Gromov-Witten invariants of the quintic to the complex geometry of its mirror family of Calabi-Yau threefolds. While there is still no direct mathematical explanation of this relation, the predicted relation has been proved for many projective manifolds by explicitly computing both sides. In this part, I will present Givental's proof for projective hypersurfaces.

[CDGP] P. Candelas, X. de la Ossa, P. Green, and L. Parkes, *A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory*, Nuclear Phys. B359 (1991), 21-74.

[MirSym] K. Hori, S. Katz, A. Klemm, R. Pandharipande, R. Thomas, C. Vafa, R. Vakil, and E. Zaslow, Mirror Symmetry, Clay Math. Inst., AMS 2003, Chapters 25-30.

[Z] A. Zinger, *Notes on Mirror Symmetry*, Chapter 4.

Part IV. *Genus 0 mirror symmetry for projective hypersurfaces in Grassmannians.*

This is a more sophisticated and general treatment of the same kind problem as in Part III.

[K] B. Kim, *Quantum hyperplane section theorem for homogeneous spaces*, Acta Math. 183 (1999), no. 1, 71-99.

[BCFK] A. Bertram, I. Ciocan-Fontanine, and B. Kim, *Two proofs of a conjecture of Hori and Vafa*, Duke Math. J. 126 (2005), no. 1, 101-136.