MAT 531: Topology&Geometry, II
Spring 2006

Problem Set 11
Due on Thursday, 5/4, in class

Problem 1

Suppose M and N are smooth oriented compact connected n-manifolds. If $f\colon M \longrightarrow N$ is a smooth map, the `degree of` f is the number $\deg f \in \mathbb{R}$ such that

$$\int_M f^*\omega = (\deg\, f)\cdot \int_N \omega \qquad \forall\, \omega \in E^n(N).$$

This number is well-defined (i.e. exists) and is an integer (you should be able to prove these two claims, but you can assume them for this problem).

(a) Show that if $f\colon M \longrightarrow N$ and $g\colon N \longrightarrow X$ are smooth maps between smooth oriented compact connected n-manifolds, then

$$\deg(g\circ f) = (\deg\, g)\cdot(\deg f).$$

(b) Show that if $f\colon M \longrightarrow N$ is a covering projection between smooth oriented compact connected n-manifolds, then $\deg f$ is the degree of f as a covering map (i.e. the number of elements in each fiber).
(c) Show that if $f\colon M \longrightarrow N$ is a smooth map of degree one, then it induces a surjective homomorphism between the fundamental groups of M and N.

Note: The last part of this problem uses a fact that has been stated previously and will be proved in class on Tuesday, 5/2. You can nearly complete the solution to this problem without this fact.