MAT 531: Topology&Geometry, II
Spring 2006

Overview

- Smooth manifolds, tangent vectors, differentials, immersions, etc.
  - PS1 #1-4c; PS2 #1-3
- Vector Bundles
  - PS1 #4d-6; PS2 #4; PS4 #4; PS6 #4a
- Differentials, Inverse FT, Slice Lemma, Implicit FT (I&II)
  - PS3 #1-3; MT #2
- Flows of Vector Fields, Lie Bracket, Lie Derivative
  - PS3 #4-6; PS4 #1,2; PS5 #3,4; PS6 #7a; MT #1
- The Differential $d\colon E^p(M) \longrightarrow E^{p+1}(M)$, Frobenius Theorem (I&II), Strong Slice Statement
  - PS4 #4; PS5 #1,2,5; MT #3
- de Rham Cochain Complex, Poincare Lemma, Stokes' Theorem (I&II), and Group Actions
  - PS6 #1,2,6,8; PS6 #5b; PS7 #5; MT #4; PS10 #3
- Orientability of Manifolds and Vector Bundles, Relations with Topology and Covering Maps
  - PS4 #3; PS6 #3-6,7bc; MT #5
- Singular Chain Complex, Hurewicz Theorem
  - PS7 #1

- (Co)Chain Complexes and (Co)homology, Duals, Coefficient Changes
  - Mayer-Vietoris for de Rham Cohomology and Singular Homology: PS7 #2-4
  - Sheafs and Čech Cohomology: PS7 #6,7; PS8 #2,3
  - Cohomology from Fine Resolutions: de Rham Theorem
  - Compactly Supported Cohomology

- Geometric Analysis
  - Differential Operators, Symbol, Elliptic Operators
  - Sobolev Lemma, Rellich Lemma, Fundamental Inequality: PS 10, #4,5

- Hodge Theory
  - Laplacian: PS3 #3; PS10 #1,2
  - Hodge Decomposition Theorem
  - Poincare Duality, Finite-Dimensionality of de Rham Cohomology
  - Kunneth Formula: PS10 #5

- de Rham Cohomology in special cases
  - $H^0_{\mathrm{de\,R}}(M)$; $H^{top}_{\mathrm{de\,R}}(M)$ (M orientable/non-orientable, compact/non-compact): PS10 #3
  - $H^*_{\mathrm{de\,R}}(\mathbb{R}^n)$, $H^*_{\mathrm{de\,R}}(S^n)$: PS7 #3