

# MAT 531: Topology&Geometry, II Spring 2011

## Problem Set 2

Due on Thursday, 2/17, in class

Give concise, but complete, solutions. The entire problem set should not require more than a few pages.

1. Chapter 1, #10 (p51)
2. Chapter 1, #7 (p50)
3. (a) For what values of  $t \in \mathbb{R}$ , is the subspace

$$\{(x_1, \dots, x_{n+1}) \in \mathbb{R}^{n+1} : x_1^2 + \dots + x_n^2 - x_{n+1}^2 = t\}$$

a smooth embedded submanifold of  $\mathbb{R}^{n+1}$ ?

(b) For such values of  $t$ , determine the diffeomorphism type of this submanifold (i.e. show that it is diffeomorphic to something rather standard).

*Hint:* Draw some pictures.

4. Show that the special unitary group

$$SU_n = \{A \in \text{Mat}_n \mathbb{C} : \bar{A}^t A = \mathbb{I}_n, \det A = 1\}$$

is a smooth compact manifold. What is its dimension?

5. Suppose  $f: X \rightarrow M$  and  $g: Y \rightarrow M$  are smooth maps that are transverse to each other:

$$T_{f(x)}M = \text{Im } d_x f + \text{Im } d_y g \quad \forall (x, y) \in X \times Y \text{ s.t. } f(x) = g(y).$$

Show that

$$X \times_M Y \equiv \{(x, y) \in X \times Y : f(x) = g(y)\}$$

is a smooth (embedded) submanifold of  $X \times Y$  of codimension equal to the dimension of  $M$  and

$$T_{(x,y)}(X \times_M Y) = \{(v, w) \in T_x X \oplus T_y Y : d_x f(v) = d_y g(w)\} \quad \forall (x, y) \in X \times_M Y.$$

## Final Exam

The current schedule for the final exams is

Tu., 5/17, MAT 535; Wed., 5/18, MAT 550

Th., 5/19, MAT 542; Fr., 5/20, MAT 531

Mon., 5/23, Calculus

Given this schedule, I'd like to suggest moving the 531 final to some other day, such as Sat-Mon 5/14-16 or Sat-Mon 5/21-23 (a later day is not possible because the semester grades are due within 48 hours, excluding weekends, of the scheduled final exam). The last class is on 5/12 and will likely be just review. I will have office hours the day before the final (whether this falls on a weekday or on a weekend day). If you are registered in this course, on 2/17 please turn in a separate sheet of paper with your thoughts on this, i.e. that you do not want to reschedule the final at all (or to a specific day) and/or list your preferences in order for the day and starting time of the exam (say, 10, 11, 12, 1, 2, and 3). It might be simplest if all of you could agree on the day and time of the final exam before 2/17.