MAT 531: Topology&Geometry, II
Spring 2010

Problem Set 9
Due on Thursday, 4/22, in class

1. Chapter 5, #17 (p217); hint: this is barely a two-liner, including justification.

2. Compute the singular cohomology of the point directly from the definition (with coefficients in a ring K).

3. Let K be any ring containting 1. For each $i \in \mathbb{Z}^+$, let $V_i = K$; this is a K-module. Whenever $i \leq j$, define

$$\rho_{ji} : V_i \longrightarrow V_j \qquad \text{by} \qquad \rho_{ji}(v) = 2^{j-i}v;$$

this is a homomorphism of K-modules. Since $\rho_{ki} = \rho_{kj}\rho_{ji}$ whenever $i \leq j \leq k$, we have a directed system and get a direct-limit K-module

$$V_\infty = \overrightarrow{\lim_{\mathbb{Z}^+}} V_i = \lim_{i \longrightarrow \infty} V_i\,.$$

   (a) Suppose $2 = 0 \in K$ (e.g. $K = \mathbb{Z}_2$). Show that $V_\infty = \{0\}$.

   (b) Suppose 2 is a unit in K (e.g. $K = \mathbb{R}$). Show that $V_\infty \approx K$ as K-modules.

   (c) Suppose 2 is not a unit in K, but $2 \neq 0 \in K$, and K is an integral domain (e.g. $K = \mathbb{Z}$). Show that the K-module V_∞ is not finitely generated.

   Note: if you prefer, you can do the *e.g.* cases; this makes no difference in the argument.

Exercise (*figure this out, but do not hand it in*): Chapter 5, #20 (p217).