MAT 401: Undergraduate Seminar

Introduction to Enumerative Geometry

Fall 2008

Homework Assignment II

Written Assignment due on Thursday, 9/18, at 11:20am in Physics P-117

(or by 9/18, 11am, in Math 3-111)

Chapter 2, #1,5,6

Please aim to make your solutions as concise and to the point as possible.

Discussion Problem for 9/18

Duality for Conics

Let $n_2(i)$ be the number of plane conics that are tangent to i general lines and pass through $5-i$ points, with $i=0,1,\ldots,5$. It is stated at the end of Chapter 2 that

i	0	1	2	3	4	5
$n_2(i)$	1	2	4	4	2	1

An argument for the numbers $n_2(i)$ for $i=0,1,2$ is given in the book. The aim of this discussion problem is to obtain the remaining numbers by showing that

$$n_2(i) = n_2(5-i). \qquad (*)$$

Part I: Chapter 2, #8 (∼25 mins)

Part II: Recall from the first discussion session and Chapter 2 that a line in $\mathbb{C}P^2$ is also a point in the dual projective plane, $(\mathbb{C}P^2)^* \approx \mathbb{C}P^2$. If $C \subset \mathbb{C}P^2$ is a smooth conic, there is a well-defined tangent line $\tau_C(z) \in (\mathbb{C}P^2)^*$ at each point $z \in C$. Show that the map

$$\tau_C : C \longrightarrow (\mathbb{C}P^2)^*, \qquad z \longrightarrow \tau_C(z),$$

is injective and is a homeomorphism (or at least a bijection) onto a smooth conic C^* in $(\mathbb{C}P^2)^*$. Furthermore, the image of the map

$$\tau_{C^*} : C^* \longrightarrow ((\mathbb{C}P^2)^*)^* = \mathbb{C}P^2$$

is the original conic C (i.e. dualizing twice gets us back to where we started). (∼25 mins)

Part III: Prove the identity $(*)$ (∼10 mins)

On Tuesday, 9/16, please volunteer to discuss one of the three parts on Thursday, 9/18.