MAT 324: Real Analysis, Fall 2017
Basic Review Questions II

1. Let $(X, \mathcal{F})$ be a measurable space and $f_1, f_2, \ldots : X \longrightarrow \overline{\mathbb{R}}$ be a sequence of measurable functions. Show that the following sets are measurable:

   (a) $\{x \in X : \lim_{n \longrightarrow \infty} f_n(x) = \infty\}$ and $\{x \in X : \lim_{n \longrightarrow \infty} f_n(x) = -\infty\}$

   (b) $\{x \in X : \lim_{n \longrightarrow \infty} f_n(x) \text{ exists and is finite}\}$

2. Define what it means for a subset $A \subset \mathbb{R}$ to be null in the usual sense. Show that a countable union of null subsets is a null subset.

   (a)

   (b)

3. Give the usual definition of outer measure on $\mathbb{R}$. Show that it is equivalent to the same definition using only open intervals.

   (a)

   (b)

4. If μ is a measure on $(X, \mathcal{F})$, how can one construct other measures on $(X, \mathcal{F})$? Prove your statements.

5. Let $(X, \mathcal{F}, \mu)$ be a measure space. What do $\mathcal{L}^p(X)$ and $L^p(X)$ mean? Why do we usually require that $p \in [1, \infty]$? Give example for why $p < 1$ is usually excluded?

6. What is special about $L^2(X)$? Give examples of why the other $L^p(X)$ do not have this property.
   (a)
   (b)
   (c)
7. What is the relation between $L^p(X)$ and $L^q(X)$? Do not forget the $p = \infty$ possibility. Give statements and examples of non-implications.
   (a)
   (b)
   (c)
8. If $f \in L^p(X)$, what conditions would ensure that $f \in L^q(X)$ with $q \neq p$? Give at least three sets of different sufficient conditions.
   (a)
   (b)
   (c)
9. What does convergence of a sequence $f_1, f_2, \ldots$ in $L^\infty(X)$ have to do with pointwise and uniform convergence (including a.e.) and vice versa? Give statements and examples of non-implications.
   (a)
   (b)
   (c)
10. What does convergence of a sequence/subsequence $f_1, f_2, \ldots$ in $L^p(X)$ with $p \neq \infty$ have to do with pointwise convergence? Give statements and examples of non-implications.
   (a)
   (b)
   (c)