

MAT 324: Real Analysis, Fall 2017

Bonus Homework Assignment

Please read carefully Sections 4.6-4.8 in the textbook. We'll start on Chapter 5 on Tuesday, 10/24.

Bonus Problem Set (**due in class on Tuesday, 10/24**): Problems 1 and 2, below and on the back.

The total score for Problems 1 and 2 will be added to the higher of your PS1 and PS2 scores. Each *completely correct and completely complete* answer in Problem 1 *legibly written in the space provided for the question* is 1 point; nothing otherwise. Problem 2 has two distinct parts, which you need to clearly identify. Each part is worth up to 5 points, with partial credit awarded, based on the answer provided on the back of this sheet (which can be continued on one extra attached sheet). Fresh copies of this assignment can be printed off the course website. If you want to typeset your answers in latex, email me for the latex source of this assignment.

Problem 1 (10pts)

Complete the following definitions.

- (a) An equivalence relation on a set X is a subset $A \subset X^2$ such that
- (b) A metric space is a pair (X, d) , where X is a set and $d: X^2 \rightarrow \mathbb{R}^{\geq 0}$ is a function such that
- (c) A map $f: (X, d) \rightarrow (X', d')$ between two metric spaces is an isometry if

Let (X, d) be a metric space.

- (d) A sequence of points $x_n \in X$ converges to a point $x \in X$ if
- (e) The closure of a subset $Y \subset X$ is the subset $\bar{Y} \subset X$ consisting of the points $x \in X$ such that
- (f) A subset $Y \subset X$ is dense if
- (g) A sequence of points $x_n \in X$ is Cauchy if
- (h) The metric space (X, d) is complete if
- (i) A completion of (X, d) is a complete metric space (\bar{X}, \bar{d}) such that
- (j) Two completions (\bar{X}, \bar{d}) and (\bar{X}', \bar{d}') of (X, d) are equivalent if

Problem 2 (10pts)

Show that every metric space (X, d) admits a unique completion, up to equivalence.