MAT 324: Real Analysis, Fall 2017
Homework Assignment 1

Please read carefully Chapter 1 and Sections 2.1 and 2.2 of the textbook, prove all propositions and do all exercises you encounter along the way, and write up clear solutions to the written assignment below. The exams in this class will be based to a large extent on these propositions, exercises, and assignments.

Problem Set 1 (**due in class on Thursday, 9/7**): Problems 1-5 below

Please write your solutions legibly; the TA may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first) and HW number in the upper-right corner of the first page.

Problem 1

Is the function

$$f(x)=\sum_{n=0}^{\infty}2^{-n}\sin(2^n x)$$

Riemann integrable on $[0,2\pi]$? Justify your answer.

Problem 2

Show that the set

$$A=\big\{x\in[-1,1]:\big|\sin(nx)\big|\le\frac{1}{n}\ \forall\, n\in\mathbb{Z}^+\big\}$$

has measure 0.

Problem 3

Let $A\subset[0,1]$ be a null subset. Show that

$$B\equiv\big\{x^2: x\in A\big\}$$

is also a null subset. Is the conclusion still true if $A\subset\mathbb{R}$? Justify your answer.

Problem 4

In Definition 2.3 in the textbook, the set Z_A involves sums taken over sequences of intervals I_n of every possible type (closed, open, open on lower/upper end and closed on upper/lower end). Show that using only one of these four kinds of intervals in the definition of Z_A would not change the definition of $m^*(A)$. This means that you need to establish 4 statements (e.g. using *open interval* in place of *interval* in the definition of Z_A would not change the answer, etc.)

Problem 5

Let $F:[0,1]\longrightarrow[0,1]$ be the Lebesgue function defined at the top of p20 in the textbook. Show that $F(0)=0$, $F(1)=1$, and F is non-decreasing, continuous, and constant on each open interval removed in the construction of the Cantor set on p19 and takes a null set to a set of outer measure 1 (the book contains an outline for justifying these statements).