# MAT 324: Real Analysis, Fall 2017
## Final Exam Information

The final exam will take place on Friday, December 15, 11:15am-1:45pm in ESS 181. It will cover Chapters 1-6 (except for Sections 2.6, 3.5, 4.7, 5.4, and 6.5) and Sections 7.2 and 8.1.

You will be given an exam cover sheet with *answer-only* Problems 1 and 2, a separate sheet with Problems 3-9, and printer paper to write your *solutions* for Problems 3-9 on; the *answers* to Problems 1 and 2 are to be written in the space provided on the cover sheet. Please start your solution to each problem 3-9 on a new page. You do not need to copy the statements of the problems, but please mark clearly what problem you are solving. Please hand in your solutions in order, staple them to the back of the exam cover sheet (stapler provided), and write your name at the top of the cover sheet in the space provided. Please do not hand in anything you do not want to be graded; you can keep the sheet with Problems 3-9.

The final exam will be roughly 50% longer than the midterm, which will give you roughly 30 minutes extra at the pace required for the midterm. So, there should be less time pressure. The exam will be out of 150 points, i.e. one point per minute, which might help with the timing. Problem 2 will consist of 10 true/false 1-point questions concerning basic notions and key statements in the course. While the questions will hopefully not require much thought, please do read them carefully. As there will be no penalty for wrong answers, do not leave these unanswered before handing in your exam. Try to aim for getting as many problems (and parts of problems) done correctly, instead of writing something for every problem, especially if your primary goal now is passing this course.

You need to know the definitions and main statements from the textbook and be able to do the textbook and homework exercises and prove lemmas/propositions and not overly technical theorems from the textbook. You do not need to memorize the definitions and statements word for word, but you need to be able to convey their meaning precisely. The next page contains more details.

The office hours during the finals week will be held on the regular schedule, *except* Yuhan's hour in the office will be after his two hours at MLC (so on Wednesday 7-8pm, instead of 3-4pm); your graded HW11 should be available then. If MLC is closed on Wednesday's evening, Yuhan will be in his office for the entire three hours instead.

I will try to get the exam graded on Friday. In line with the department's policy, I will keep your final exam. If you want a scan of it, send me an email after the exam. If you would like a hardcopy, to go over it, and/or to discuss what classes to take next term and such things, please send me an email with your schedule for the few days afterwards.

**Things you should be familiar with and know how to use include:**

1. **Basic notions:**

   1a. collections of subsets of a set, field, $\sigma$-field, monotone class, generation by a collection of subsets, Borel subsets of $\mathbb{R}$ and $\mathbb{R}^n$

   1b. measure space, countable additivity and subadditivity, completeness and completion of measure space, outer measure, null subset, measurable subset, construction of measure space from outer measure, Cantor set

   1c. measurable function, Borel vs. Lebesgue measurable function, simple/step function, Lebesgue and Cantor functions

   1d. Riemann integral, Riemann's criterion, fundamental theorem of calculus, Lebesgue integral, characterization of Riemann integrable functions

2. **Main theorems:**

   2a. Interchange of limits and integration (Fatou's Lemma, Monotone Convergence, Dominated Convergence)

   2b. Lebesgue v.s. Riemann integration, proper and improper

   2c. Fubini's theorems (interchange of integrals and comparison with product integral)

   2d. Radon-Nikodym theorem (relations between different measures)

   You need to know what these say, including the assumptions. It generally helps if the relevant function $f \geq 0$, but this is not always sufficient or necessary.

3. **Functional analysis:** Cauchy sequence, complete metric space, Banach/Hilbert spaces, Hölder's and Minskowski inequalities, Beppo-Levi, $L^p$-spaces and their completeness

4. **Approximations of measurable functions:** by step functions in general, by "nice" functions and continuous functions when the domain is $\mathbb{R}^n$

5. **Key tricks:**

   5a. $a \leq b$ if $a \leq b+\varepsilon$ for all $\varepsilon > 0$

   5b. the $\varepsilon/2^n$ trick

   5c. to show that some subset of a measurable space is measurable, write it as a finite/countable union/intersection of (finite/countable intersections/unions of) obviously measurable subsets

   5d. to establish some integral identity both sides of which are *linear* on some space of functions, first check it for the indicator functions, then deduce it for simple nonnegative functions, then for nonnegative functions via approximation by an increasing sequence of simple functions and Monotone Convergence Theorem, then for integrable real-valued functions, and finally for possibly complex-valued integrable functions

   5e. to show that every element $E$ of some nice collection $\mathcal{F}$ of subsets of a set $X$ (such as a $\sigma$-field) satisfies some property, show that the subcollection $\mathcal{F}' \subset \mathcal{F}$ of the elements $E$ satisfying this property has some nice structure implying that $\mathcal{F}' = \mathcal{F}$ (e.g. $\mathcal{F}'$ is a monotone class containing a generating set for $\mathcal{F}$).