MAT 312/AMS 351: Applied Algebra
Homework Assignment 5

Written Assignment due before 11:30am, Thursday, 10/10

Please read Section 4.3 and pp200-206 before starting on the problem set.

Practice Problems (do not hand in; answers in the book): 4.3 1,2,5,7,8

Written Assignment: 4.3 3,4,6; Problem B (below)

Show your work; correct answers without explanation will receive no credit, unless noted otherwise

Please write your solutions legibly; the grader will disregard solutions that he does not find readily readable (you are encouraged to type up your solutions, especially if your handwriting is not absolutely immaculate). The problems on your solutions must appear in the assigned order; out-of-order problems will not be graded. All solutions must be stapled (no paper clips) and have your name (first name first), recitation number (R01 or R02), and HW number in the upper-right corner of the first page; otherwise, you may receive no credit.

NO late homework will be accepted

Problem B

Let G be a group and e be its identity element.

(a) Suppose $G = \{e, a\}$ consists of exactly two distinct elements. Show that $a^2 = e$.

(b) Suppose $G = \{e, a, b\}$ consists of exactly three distinct elements. Show that $a^2 = b$ and $a^3 = e$.

(c) Suppose $|G| = 4$. Show that either there exists $a \in G$ such that $G = \{1, a, a^2, a^3\}$ with $a^4 = e$ or there exist distinct $a, b \in G$ such that $G = \{1, a, b, ab\}$ with $a^2, b^2 = e$ and $ab = ba$.

Note: By (a) and (b), all groups of orders 2 and 3 are isomorphic to $(\mathbb{Z}_2, +)$ and $(\mathbb{Z}_3, +)$, respectively. By (c), every group of order 4 is isomorphic either to $(\mathbb{Z}_4, +)$ or $(\mathbb{Z}_2, +) \times (\mathbb{Z}_2, +)$. In particular, all of these groups are abelian. The smallest non-abelian group, S_3, has 6 elements.