Annals of Mathematics, 119 (1984), 639

Typesetter's correction to

"Classification of irreducible tempered representations of semisimple groups"

By A. W. Knapp and Gregg J. Zuckerman

116 (1982), 389–501

We regret that five extraneous lines were introduced and four lines deleted by error in the September, 1982 issue of the Annals. The following is the corrected page 420 of Volume 116.

Then we compute

$$\begin{aligned} \varphi^{\lambda}_{\lambda_1}\Theta(\lambda, C) &= \varphi^{\lambda}_{\lambda_1}\psi^{\lambda_1}_{\lambda}\Theta(\lambda_1, C) && \text{by (5.8)} \\ &= \sum_{w\in W(\lambda)} w\Theta(\lambda_1, C) && \text{by Theorem C.3} \\ &= |W(\lambda)|\Theta(\lambda_1, C) + j''' && (5.13) \end{aligned}$$

by (5.12). In view of (5.7), $\varphi^{\lambda}_{\lambda_1}$ and $\varphi^{\lambda'}_{\lambda'_1}$ are the same functor. Thus there is an equation analogous to (5.13) for $\varphi^{\lambda'}_{\lambda'_1}$ and (5.6) gives

$$|W(\lambda)|\Theta(\lambda_1, C) = |W(\lambda)|\Theta(\lambda'_1, C') + j''''.$$

Decompose $j'''' = p - n$ into the difference of two true characters. Then we have

$$p + |W(\lambda)|\Theta(\lambda'_1, C') = n + |W(\lambda)|\Theta(\lambda_1, C).$$

Every irreducible constituent of p and of n is in $J(\psi^{\lambda_1}_{\lambda})$, and the discrete series characters $\Theta(\lambda'_1, C')$ and $\Theta(\lambda_1, C)$ are not in $J(\psi^{\lambda_1}_{\lambda})$. Since distinct irreducible characters are linearly independent, we conclude that

$$\Theta(\lambda'_1, C') = \Theta(\lambda_1, C). \tag{5.14}$$

The characters in (5.14) are discrete series characters, and thus the element w_0 in $W(B : G^C)$ with $C' = w_0 C$ has to be in $W(B : G)$. We have seen that $\lambda' = w_0\lambda$. Thus the proof of Theorem 1.1 is complete.

6. Inversion of generalized Schmid identities

In Section 5 we used generalized Schmid identities to imbed basic characters from one cuspidal parabolic in basic characters from a different parabolic with a more noncompact A. Now we turn matters around, in order to use generalized Schmid identities to exhibit reducibility. There is an obstruction to inverting the identities, having to do with integrality.

Theorem 6.1. *Let $P = MAN$ be a cuspidal parabolic subgroup of a linear connected reductive Lie group G with compact center. Let $\mathfrak{b} \subseteq \mathfrak{k} \cap \mathfrak{m}$ be a compact Cartan subalgebra of $\mathfrak{m}$, and let α be a real root of $(\mathfrak{g}^{\mathbf{C}}, (\mathfrak{a} \oplus \mathfrak{b})^{\mathbf{C}})$. Suppose that the Cayley transform $\mathbf{d}_\alpha$ leads from the data $(\mathfrak{m}, \mathfrak{a}, \mathfrak{b})$ to data $(\mathfrak{m}^*, \mathfrak{a}^*, \mathfrak{b}^*)$ and that $\tilde{\alpha} = \mathbf{d}_\alpha(\alpha)$. Then the character*

$$\operatorname{ind}^{M^*A^*}_{P\cap M^*A^*}\Theta^{MA}(\lambda_M, C_M, \chi_M, \nu_M)$$

is the right side of a generalized Schmid identity (4.4a) *or* (4.4b) *obtained from $\tilde{\alpha}$ if and only if $\langle \nu_M, \alpha\rangle = 0$ and*

$$\chi_M(\gamma_\alpha) = (-1)^{2\langle \rho_\alpha, \alpha\rangle/|\alpha|^2}, \tag{6.1}$$