Corrections and Clarifications to
Basic Real Analysis*, *Digital Second Edition

Page 2, line −16. Change "subtraction" to "negative".

Page 2, line −15. Delete "or difference".

Page 2, line −14. Insert after "respective cuts" the clause ", and the negative of a cut r is the set of all rationals q such that there exists a rational $q' > 0$ with $-q - q'$ not in r".

Page 3, line 7. Change "Then $-x$ is" to "Then $-\sup_{x\in -E} x$ is ".

Page 8, proof of (c), line 2. Change "choose $N \geq n_{l-1}$" to "choose $N > n_{l-1}$".

Page 8, proof of (c), line 3. Change "choose $n_l > n_{l-1}$" to "choose $n_l \geq N$".

Page 8, line −1. Change "finitely many a" to "finitely many n".

Page 11, proof of Theorem 1.10, line −2. Change "$\delta_{x_{n_k}} \geq \frac{1}{2}\delta_{x'}(\frac{\epsilon}{2})$" to "$\delta_{x_{n_k}}(\epsilon) \geq \frac{1}{2}\delta_{x'}(\frac{\epsilon}{2})$".

Page 22, line −10. Change "Choose" to "Arguing by contradiction, choose".

Page 22, line −9. Change ". Then" to ", and then".

Page 22, line −8. Change "$\sup_n K_n$ in $\mathbb{R}^*$" to "$\sup_n K_n = +\infty$ in $\mathbb{R}^*$".

Page 22, line −1. Change "$1 + M_{x_0}$ is a uniform bound for the functions f_n" to "$1 + M_{x_0}$ is a (finite) uniform bound for the functions f_n, contradiction".

Page 23, line −5 of proof of (a). To simplify the notation, change "$\delta'/2$" to "δ'" in two places.

Page 24, line 6 of Example. Change "x is in the closed interval $[a, b]$ and t is in the open interval (a, b)" to "x and t are distinct members of the closed interval $[a, b]$".

Page 30, lines 2 and 3 of Remarks. For easier reading, change "$\int_b^a f\,dx = -\int_a^b f\,dx$ when $b < a$" to "$\int_a^b f\,dx = -\int_b^a f\,dx$ when $a > b$".

Page 30, proof of Lemma 1.27, lines 3 and 4. Change "the larger of the numbers of times c occurs in P_1 and P_2" to "one less than the sum of the number of times that c occurs in P_1 and the number of times that c occurs in P_2".

Page 35. lines 2 to 4. Delete the sentence "It follows ... say by M."

Page 35, line 4. Change "$|f(x)| \leq M$" to "$|f(x)| \leq M_N + 1$".

Page 39, proof of Theorem 1.35, line 5. Change "types" to "kinds" for consistency of terminology.

Page 40, line 11. Change "ϵ" to "3ϵ" in the middle, and change "3ϵ" to "4ϵ" at the right end.

Page 40, line 13. Change "3ϵ" to "4ϵ".

Page 40, line 15. Change "3ϵ" to "4ϵ" twice.

Page 40, line 16. Change "3ϵ" to "4ϵ".

Page 40, first line of last display. Change "$\big|U(P, f) - \overline{\int_a}^b f\,dx\big|$" to "$\big|\overline{\int_a}^b f\,dx - U(P, f)\big|$" for easier reading.

Page 41, line −3. Change "$\max\{\operatorname{Re} w, \operatorname{Im} w\}$" to "$\max\{|\operatorname{Re} w|, |\operatorname{Im} w|\}$" on both sides of the inequality.

Page 45, lines 12–13. Change "see examples both where the limit is identically 0 and where it is" to
"see convergent examples where this limit is identically 0 and where this limit is".

Page 45, line −5. Change "$\sum_{n=0}^\infty c_n z^n$" to "$\sum_{n=0}^\infty |c_n z^n|$".

Page 46, line −5. Change "of f" to "of f about $x = a$".

Page 48, line −5. Change "$\epsilon G(z) + \epsilon F(z)$" to "$\epsilon F(z) + \epsilon G(z)$" for parallelism with the previous line.

Page 50, just before Corollary 1.43. Insert a one-sentence paragraph saying, "With Corollary 1.42 in place, we define $a^z = e^{z \log a}$ for $a > 0$ and $z \in \mathbb{C}$."

Page 51, line −8. Change "$[0, 2\pi)$" to "$[0, 2\pi)$ and $x_1 \ge x_2$".

Page 51, lines −7 and −6. Change "$(-2\pi, 2\pi)$" to "$[0, 2\pi)$".

Page 58, line −17. Change "$|\sigma_k - s| \le \epsilon^2$" to "$|\sigma_k - s| < \epsilon^2$".

Page 60, lines −12 and −10. Change "$\delta \le x \le 1$" to "$\delta \le |x| \le 1$".

Page 61, line 1. Change "$Q(cx)$" to "$Q(cx)$ on $[0, 1]$".

Page 86, line −2 of (2). Change "the Hermitian inner product" to
"the real part of the Hermitian inner product".

Page 113, line 5 of the proof of Theorem 2.42. Change "Choosing $R = R_0$" to
"Choosing $R_0 \ge R$".

Page 190, line 5 of proof in the middle of the page. Change
"$\Big| \sum_{j=1}^m |\gamma'(t_j)|(t_j - t_{j-1}) - \int_{a'}^{b'} |\gamma'(t)\, dt \Big| < \varepsilon$" to
"$\Big| \sum_{j=1}^m |\gamma'(t_j)|(t_j - t_{j-1}) - \int_{a'}^{b'} |\gamma'(t)|\, dt \Big| < \varepsilon$".

Page 190, line 10 of proof in the middle of the page. Change
"$\Big| \sum_{j=1}^m |\gamma'(t_j)|(t_j - t_{j-1}) - \int_{a'}^{b'} |\gamma'(t)\, dt \Big| < \epsilon$" to
"$\Big| \sum_{j=1}^m |\gamma'(t_j)|(t_j - t_{j-1}) - \int_{a'}^{b'} |\gamma'(t)|\, dt \Big| < \epsilon$".

Page 191, two lines below "(††)" change the left member of the inequality from
"$\Big|\ell(\gamma_{[a',b']}) - \int_{a'}^{b'} |\gamma'(t)\, dt\Big|$" to "$\Big|\ell(\gamma_{[a',b']}) - \int_{a'}^{b'} |\gamma'(t)|\, dt\Big|$".

Page 201, line 1. Change "q by a piecewise" to "p by a piecewise".

Page 554, proof of Proposition 11.17. The phrase "it follows that $\mathcal{B}_0(X) \times \mathcal{B}_0(Y) \subseteq \mathcal{B}_0(X \times Y)$" in the first sentence of the proof needs some justification. Thus replace the first sentence of the proof with the following paragraphs:

"If K_X and K_Y are compact G_δ's in X and Y, then $K_X \times K_Y$ is a compact G_δ in $X \times Y$. Consequently $K_X \times K_Y$ is a member of $\mathcal{B}_0(X \times Y)$. More generally let K_X and L_X be compact G_δ's in X with $L_X \subseteq K_X$, and let K_Y and L_Y be compact G_δ's in Y with $L_Y \subseteq K_Y$. Then $(K_X - L_X) \times (K_Y - L_Y)$ is a member of $\mathcal{B}_0(X \times Y)$.

"We need to observe that if $\mathcal{K}_0(X)$ is the collection of all finite disjoint unions in X of sets $K - L$, where K and L are compact sets of type G_δ in X with $L \subseteq K$, then $\mathcal{K}_0(X)$ is a ring of sets. To verify this observation, one argues exactly as in the proof of Lemma 11.2. A similar observation applies to Y, and we conclude thet every set $E \times F$ with E in $\mathcal{K}_0(X)$ and F in $\mathcal{K}_0(Y$ is in $\mathcal{B}_0(X \times Y)$.

"Fix a member E of $\mathcal{K}_0(X)$. The class of subsets F of Y for which $E \times F$ is in $\mathcal{B}_0(X \times Y)$ is a monotone class that contains all members of $\mathcal{K}_0(Y)$. Since $\mathcal{K}_0(Y)$ is a ring of sets, the Monotone Class Lemma (Lemma 5.43) allows us to conclude that the class contains $\mathcal{B}_0(Y)$. A second application of the Monotone Class Lemma shows that every set $E \times F$ with E in $\mathcal{B}_0(X)$ and F in $\mathcal{B}_0(Y)$ is in $\mathcal{B}_0(X \times Y)$. We can therefore conclude that $\mathcal{B}_0(X) \times \mathcal{B}_0(Y) \subseteq \mathcal{B}_0(X \times Y)$."

Page 593, line 10. Change "closed sets in Y" to "closed sets in $\mathbb{R}$".

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